

Discrete Mathematics --- Day 9 --- September 15, 2004

#12 from homework.

P	Q	R	Target Column ???	$P \wedge Q \wedge R$	$P \wedge \neg Q \wedge R$	$\neg P \wedge Q \wedge R$	\wedge	\wedge	\wedge
T	T	F	F	T	F	F	-	-	-
T	T	F	F	F	F	F	-	-	-
T	F	T	T	F	T	F	-	-	-
T	F	F	F	F	F	F	-	-	-
F	T	T	T	F	F	T	-	-	-
F	T	T	T	F	F	F	T	-	-
F	F	T	T	F	F	F	-	T	-
F	F	T	T	F	F	F	-	-	T

??? = $(P \wedge \neg Q \wedge R)$ or $(\neg P \wedge Q \wedge R)$ or $(\neg P \wedge Q \wedge \neg R)$ or $(\neg P \wedge \neg Q \wedge R)$ or $(\neg P \wedge \neg Q \wedge \neg R)$

* Theoretically any circuit is buildable

Negating Quantified Expressions

$\forall (b) G(b)$ b: is a good boy
 G(x): x does fine

$$\neg \forall (b) G(b) = \exists b \neg G(b)$$

$$\neg \exists (b) G(b) = \forall (b) \neg G(b)$$

- (1) The negation of $\forall (x) P(x)$ is $\exists (x) \neg P(x)$
- (2) Consider the following: $\forall (x) \neg P(x)$

$$[\forall (x) \neg P(x)] \rightarrow [\exists (x) \neg P(x)]$$

So (2) is stronger than (1) in the sense that (2) implies (1)

Two new logical operations

\oplus exclusive "OR" and \leftrightarrow "If and only if"

p	q	$p \oplus q$	$\neg(p \oplus q)$	*is false if both variables are true or if both variables are false
T	T	F	T	
T	F	T	F	
F	T	T	F	
F	F	F	T	

p	q	$p \leftrightarrow q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	
T	F	F	
F	T	F	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
F	F	T	

T T T T T T T T
 T F F F F T T T
 F T T F T F F F
 F T F T F T F F

*True if p and q have the same value

*False if p and q have different values

$$\neg(p \oplus q) \equiv p \leftrightarrow q$$

$$\neg(p \leftrightarrow q) \equiv p \oplus q$$

$$\neg[(p \oplus q) \rightarrow (p \leftrightarrow q)]$$

$$\equiv (p \oplus q) \wedge (p \oplus q)$$

$$\equiv p \oplus q$$