Discrete Mathematics --- Day 8 --- September 13, 2004

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Proving Equivalencies with Truth Tables:

Put your variables in alphabetical order like so.

P	**Q**	**R**	**¬R**	**¬R V Q**	**R V P**	**(R V P) Λ (¬R V Q)**
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	T	F	T	T	F	F
F	F	T	F	F	T	F
F	F	F	T	T	F	F

Logical Equivalencies:

$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$

Proof:

P	Q	P → Q	¬P	¬Q	¬Q → ¬P
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

DeMorgan's Rules:

(R V P) Λ (¬R V Q) ≡ P V Q
¬(P Λ Q) ≡ ¬P V ¬Q
¬(P V Q) ≡ ¬P Λ ¬ Q

Negating Compound Propositions:

Ground rule:
Only have a ¬ attached directly to a variable , eg ¬Q .
You CAN'T have something like ¬(Q V P).

Here's some examples of negating compound Propositions:

"It's either raining or snowing" … You're a liar if both are false.
So the negation of it would be like this:
Raining = P, Snowing = Q

So the negation of (P V Q) is:
¬(P V Q)
≡ ¬P Λ ¬Q

SPECIAL NOTE:
If you negate something twice, you end up with the same thing as you started,
Here's an example:

What we start with:
P V Q

Negation 1:
¬(P V Q)
¬P Λ ¬Q
Negation 2:
¬(¬P Λ ¬Q)
P V Q

As you can see, we are back where we started.

Now lets look at negating implications:

First rule of thumb is when negating implications, the first part stays the same. Here's an example of negating an implication:

¬[(¬P V Q) → (S → ¬T)]
≡ (¬P VQ) Λ ¬(S → ¬T)
≡ (¬ P V Q) Λ (S Λ T)

Notice there are no "→" left , and only the ¬ are with variables. So we are finished!