Discrete Mathematics --- Day 5 --- September 3, 2004

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In class this day we went over the homework that was due today, the handout that was given problems 1-7.

Here are the problems right here:

1.) **Find two examples of "resolution" syllogism in the play. The conclusions should be disjunctions of terms which are not negated. Identify the propositions in each case, and represent the syllogism symbolically.**

For this problem, you had to use the other handout Hochberg gave us titled "A Symbolic Summary of the Play". For those of you who do not have it or lost it here it is:

Statements and Predicates

P(x)	Person x never got his or her feet wet
Q(x)	Person x was in the faculty lounge
R	The secretary was in the principal's office
S(x)	Person x was gnawed by rats
T(x)	Person x smelled like Limburger
U(x)	Person x had changed his or her socks
W	The secretary was wearing shoes
Z(x)	Person x was clawed by cats

Assertions and their logical expressions

P(x) V U(x)	Person x either changed his or her socks or never got his or her feet wet.
Q(x) V ¬U(x)	Either didn't change his or her socks or was in the faculty lounge.
¬W V R	Either the secretary was not wearing shoes or she was in the principal's office
T(x) → Z(x)	If person x smelled like Limburger, then person x was clawed by cats.
Q(x) → T(x)	If person x was in the faculty lounge, then person x smells like Limburger.
W V ¬P(secretary)	Either the secretary was wearing shoes or her feet got wet.
¬(S(secretary) V Z(secretary))	The Secretary was neither gnawed by rats nor clawed by cats.

Now back to number one, you had to find 2 examples of "resolution" in the play.
If you forgot what the resolution formula was, here it is:

P V Q
¬P V R
____

Therefore, Q V R
So what two disjunctions in the table fit into this formula?

Here is one:

Person x either changed his or her socks or never got his or her feet wet.
Either didn't change his or her socks or was in the faculty lounge.

Therefore, Person x either never got his or her feet wet, or was in the faculty lounge.

And here it is symbolically:

P(x) V U(x)
Q(x) V ¬U(x)

____
Therefore, P(x) V Q(x)

Here is another one:

Either the secretary was wearing shoes or her feet got wet.
Either the secretary was not wearing shoes or she was in the principal's office.

Therefore, either the secretary's feet got wet, or she was in the principal's office.

Symbolically:

W V ¬P(secretary)
¬W V R

____
Therefore, ¬P(secretary) V R

2.) Identify two instances of modus tollens in today's play.

The formula for modus tollens is:

P → Q
¬Q

____
Therefore, ¬P

Here is one instance of MT (modus tollens) from the play:

If the Secretary was in the faculty lounge, then she smelled like Limburger.
The Secretary didn't smell like limburger.

Therefore, the Secretary wasn't in the faculty lounge.

Q(secretary) → T(secretary)
¬T(secretary)

____
Therefore, ¬Q(secretary)

Here is another instance of MT from the play:

If the Secretary smelled like Limburger, then she got clawed by cats.
The Secretary was not clawed by cats.

Therefore, the Secretary did not smell like Limburger.
Symbolically it would look like this:

$T(secretary) \rightarrow Z(secretary)$
$\neg Z(secretary)$

____
Therefore, $\neg T(secretary)$

3.) Identify an instance of hypothetical syllogism in today's play.

The formula for HS (hypothetical syllogism) looks like this:

$P \rightarrow Q$
$Q \rightarrow R$

____
Therefore, $P \rightarrow R$

This I believe is an easy formula to remember. There are several instances in the play but here is one of them.

If person x was in the faculty lounge, then person x smells like Limburger.
If person x smelled like Limburger, then person x was clawed by cats.

Therefore, if person x was in the faculty lounge, then person x was clawed by cats.

$Q(x) \rightarrow T(x)$
$T(x) \rightarrow Z(x)$

____
Therefore, $Q(x) \rightarrow Z(x)$

Hang in there guys, we still got 4 more problems to do.

4.) Suppose that $P \rightarrow Q$, $Q \rightarrow R$, and $R \rightarrow S$ are all true. What assertion about S would lead us to conclude that P is false? How do you prove your claim?

To solve a problem like this, it helps to write it down and number it like so:

1. $P \rightarrow Q$
2. $Q \rightarrow R$
3. $R \rightarrow S$
4.

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5. Therefore, $\neg P$

So when you look at it that way, we have to find out what fits in that number 4 slot, and if you look closer you can see that Hypothetic Syllogism can be used a few times to narrow down the field a little bit. So now you end up with:

$P \rightarrow S$

Therefore, $\neg P$

What has to be true about S to make P false?

Well, by the rule of MT, if the conclusion is false, then the premise HAS to be false. So for P to be false, S has to false as well. So you end up with something like this:

1. P → Q
2. Q → R
3. R → S
4. ¬S

Therefore, ¬P

5.) Assuming that ¬A → B, A → (C → D), and E → ¬B are all true, show that E → (C → D) is true, or find values for each of the variables so that the premises are true but the conclusion is false.

So like we did in the other problem, lets write it down and number the lines.

1. ¬A → B
2. A → (C → D)
3. E → ¬B

So looking at it this way, what things can we conclude?

4. Taking the <u>contrapositive</u> of number 3 will give us B → ¬E.

5. Taking the <u>contrapositive</u> of number 1 will give us ¬B → A.

6. Using H.S. with number 3 and 5 we can conclude that E → A.

7. Using H.S. with number 6 and 2 we can conclude that E → (C → D)

So after all of that work you should come up with something that looks like this:

1. ¬A → B
2. A → (C → D)
3. E → ¬B

Therefore,

4. B → ¬E
5. ¬B → A
6. E → A
7. E → (C → D)

6.) Assuming that (A → B) → C, D → E, ¬D → ¬C, and ¬E are all true, show that ¬(A → E) is true.

Here is a very tough problem that a lot of the class had trouble on. Well let us start out by putting down what we have and numbering them.

1. (A → B) → C
2. D → E
3. ¬D → ¬C
4. ¬E

Therefore,

5. ¬(A → E)

So when you look at the conclusion, it is saying that you have to prove that A → E is false, and how do you do that? You have to show that A is true and E is false. That is the only way you can show that an implication is false.

Well they give you an easy hint by saying that E is false already on line 4. So now all you have to do is prove that A is true.

6. By the M.T. rule on line 2 with line 4, you can conclude that $\neg D$ is true.

7. By using the M.P. rule on line 3 and 6, you can conclude that $\neg C$ is true.

8. By using the M.P. rule on line 1 and 7, you can conclude that $\neg(A \rightarrow B)$ is true.

For $A \rightarrow B$ to be false, A has to be true. You are done with the problem because you have shown that A is true and E is false.

It should all look together like this:

1. $(A \rightarrow B) \rightarrow C$
2. $D \rightarrow E$
3. $\neg D \rightarrow \neg C$
4. $\neg E$

Therefore,

5. $\neg(A \rightarrow E)$
6. $\neg D$
7. $\neg C$
8. $\neg(A \rightarrow B)$

7.) Assume that "If I love you, then I will give you flowers" is true. Which of the following must also be true:

a.) If I give you flowers, then I love you.

b.) If I don't give you flowers, then I don't love you.

c.) If I don't love you, then I don't give you flowers.

If you say that an assertion must be true, justify your answer. If not, explain circumstances under which the assertion might be false.

a. This does not have to be true because it is a converse of the original formula. Implications do NOT imply that its converse is true.
   If $A \rightarrow B$ is true, it doesn't mean that $B \rightarrow A$ is true.
b. This is true because it is a contrapositive of the original formula. Implications DO imply that its contrapositive is true.
   If $A \rightarrow B$ is true, then $\neg B \rightarrow \neg A$ is also true as well.
c. This is false because it is an inverse of the original formula. Implications do NOT imply that its inverse is true.
   If $A \rightarrow B$ is true, it doesn't mean that $\neg A \rightarrow \neg B$ is true.

Near the end of class, Hochberg put a problem up on the board and asked us to "solve" it.

$E \rightarrow C \rightarrow D$

After many different answers he told us that the answer was UNDEFINED. He explained this using 2 analogies.

Analogy #1

$3 + 4 + 5 = 12$
$(3 + 4) + 5 = 12$
$3 + (4 + 5) = 12$

No matter what order you add the numbers together in, you will ALWAYS get 12. Addition IS associative.

Analogy #2

3 – 4 – 5 = ?
(3 – 4) – 5 = -6
3 – (4 – 5) = 4

With subtraction, on the other hand, the solution depends on the order you subtract from. Subtraction is NOT associative.

Now, back to the original problem:

$E \rightarrow C \rightarrow D$

He said that this was undefined because it could be $(E \rightarrow C) \rightarrow D$ or $E \rightarrow (C \rightarrow D)$. Logic is NOT associative. If you run into a problem like this, then something wrong has happened.

That is all we went over in class this day.