Discrete Mathematics – Day 35 – November 17, 2004

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Choose Polynomials

A set with n elements has $\binom{n}{k}$ subsets of size k.

Examples:

$$\binom{n}{2} = \frac{1}{2}n(n-1)$$

$$\binom{n}{3} = \frac{1}{6}n(n-1)(n-2)$$

$$\binom{n}{4} = \frac{1}{24}n(n-1)(n-2)(n-3)$$

Basically, the formula goes as such:

$$\binom{n}{k} = \frac{1}{k!}n(n-1)_{k-k}(n-2)_{k-(k-1)}...(n-(k-1))_{k-1}$$

In its expanded form using $k = 3$

$$\binom{n}{3} = \left(\frac{n!}{3!(n-3)!}\right) = \frac{n(n-1)(n-2)(n-3)(n-4)...3\times2\times1}{3\times2\times1\times(n-3)(n-4)...3\times2\times1} = \frac{n(n-1)(n-2)}{6}$$

Anagram Problems

Find all the anagrams of the word BLUFF without the F's together.
The trick to this problem is to treat the two F's as a single letter and subtract that from all the possible anagrams.

$$|A| = |U| - |\overline{A}|$$

When the two F's are combined, there are now 4 letters. There are $\frac{5!}{2!}$ total possibilities, when you subtract the 4! anagrams you get 36 total anagrams without 2 F's together.

When finding the anagrams of AABBCDEFG without the A's or the B's together, you can use the same technique. The only problem with this is that you have to add back the number of anagrams where both the A's and the B's touch because you would have subtracted them twice. The final equation for these anagrams is:

$$\frac{9!}{2!2!}-\frac{8!}{2!}-\frac{8!}{2!}+7!=55{,}440$$

<u>Geometric Sum</u>

Find the product: $2+6+18+54+....+(2\times 3^{15})$

First, you must find the ratio between the numbers. In this case, the ratio is 3. The function for this is as follows:

$$\frac{Next-First}{Ratio-1} \text{ so } \frac{(2\times 3^{16})-2}{3-1}$$