Discrete Mathematics - Day 33 - November 12, 2004

(Exam II   NEXT FRIDAY)

Suppose you expand $([3x]+[2y])^{19}$

Q: What's coefficient of $x^{10}y^{9}$?

The term to consider is $\binom{19}{10}[3x]^{10}[2y]^{9}$

$$=\binom{19}{10}3^{10}\cdot x^{10}\cdot 2^{9}\cdot y^{9}$$

$$\frac{19!}{10!9!}\cdot 3^{10}\cdot 2^{9}\cdot x^{10}\cdot y^{9}$$

Coefficient

<u>Sample Test Question</u>

If we expand $(2a + b - 3c)^5$, what is the coefficient of a^2b^2c? (Give you answer as an integer)

$$\frac{5!}{2!\cdot 2!\cdot 1!} \qquad [2a]^2\,[b]^2\,[-3c]^1$$

$$\frac{5\cdot 4\cdot 3\cdot 2}{2\cdot 2}\cdot 2^2\cdot 1\cdot(-3) \qquad a^3b^2c$$

Coefficient = **-360**

$$\sum_{\substack{1<a<b<5\\ a,b\in\mathbb{Z}}} ab = 2\cdot 3+2\cdot 4+3\cdot 4=26$$

What pairs a, b made the predicate "1 < a < b < 5" true ?

Values of a	Values of b
2	3, 4
3	4

$$\sum_{\substack{x^2+y^2=65 \\ x,y\in\mathbb{Z}}} (3x+y)$$

$8^2+1^2=65$
$7^2+4^2=65$

x	y
8	1
8	-1
-8	1
-8	-1
1	8
1	-8
-1	8
-1	-8
7	4
7	-4
-7	4
-7	-4
4	7
4	-7
-4	7
-4	-7

The sum of the 3x+y terms over these pairs of x and y will be 0, since we have a negative term for each corresponding positive term.

Here is another question which you might see on the test: Suppose I change the universe of discourse for x and y to the natural numbers, so that the negative numbers won't cancel out the positive ones anymore?

$$\sum_{\substack{x^2+y^2=65 \\ x,y\in\mathbb{N}}} (3x+y)$$

$= (3\cdot 8+1)+(3\cdot 1+8) + (3\cdot 7+4)+(3\cdot 4+7)$
= 25+11+25+19
= 80

To add a sum over a predicate:

- Find all values that satisfy the predicate
- Add the expression up for those values

$\sum_{P} p^2$ $= 2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2$

P prime 4+9+25+49+121+169+289
$1 \leq p \leq 20$ =1027 (Probably)! lol

1 is **NOT PRIME**

Here are some sums with No Thinking Required

Arithmetic Sums: Successive terms have constant differences.

EX: 9+13+17+21+25+29+33+37+41+45 (Difference between these is +4)

$$\left(\frac{9+45}{2}\right)\cdot 10 = 270$$

Geometric Sums: Successive terms that have a constant ratio.

EX: 9+18+36+72+144+288+576+1152 (ratio 1:2 $\therefore$ doubling)

2304 – 9 = 2295

Arithmetic Sums: sum is the average term times the # of terms

$$=\left(\frac{firstterm + lastterm}{2}\right) \bullet \text{ \# of terms}$$

Geometric Sums:

$$\frac{\text{(take next term) – (first term)}}{\text{ratio - 1}}$$

Proof for geometric sums:
Consider a geometric sum

$a + ar + ar^2 + ar^3 + \ldots + ar^n$

First term is a
Ratio = r

S = sum = at $ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar'$

$rs - s = ar^{n+1} - a$

$s(r - 1) = ar^{n+1} - a$

$s = \dfrac{ar^{n+1} - a}{r - 1}$ as long as $r \neq 1$

Sample Test Question:

$$\sum_{2=3}^{17} 2 \cdot (-4)^i$$

$= 2 \cdot (-4)^3 + 2(-4)^4 + \ldots + 2(-4)^{17}$

geometric
first = $2 \cdot (-4)^3 = -128$

ratio = -4

next $2 \cdot (-4)^{18}$

Sum = $\dfrac{2 \cdot (-4)^{18} - (-128)}{-4 - 1}$

= $\dfrac{2 \cdot (-4)^{18} + 128}{-5}$