Discrete Mathematics – Day 32 – November 10, 2004

Binomial Theorem (this theorem was introduced in the previous notes)

$$(s + t)^n = \sum_{i=0}^{n} \binom{n}{i} s^{n-i} t^i$$

for $n \geq 0$

Written another way:

$$(s + t)^n = \sum_{0 \leq i \leq n}^{n} \binom{n}{i} s^{n-i} t^i$$

Set Builder Notation: (an example of how this and this way of writing the Binomial Theorem are alike)

$\{ x \mid P(x) \}$

where P(x) is the predicate

eg. $\{ x \mid x^2 + 4 = 20 \}$
$= \{ 4 , -4 \}$

$\{ x \in \boldsymbol{R} \mid x^2 - 4 = 0 \}$
$= \{ 2, -2 \}$

This same concept can be viewed as Sum Builder Notation:

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$\sum f(x)$
P(x) = predicate

The sum of all function values for the *x*'s for which P(*x*) is true.

$\sum i^2$
$1 \leq i \leq 17$ this would look like $1^2 + 3^2 + 5^2 + 7^2 + 9^2 + 11^2 + 13^2 + 15^2 + 17^2$
When 'i' is odd

$$= \sum_{i=1}^{8} (2i + 1)^2$$

Here we have a predicate with two values

$$\sum_{\substack{a+b=4 \\ a,b\in N}}^{8} \binom{4}{b} x^a y^b \quad x^a y^b$$

now, to find all the parts that add up to 4 and plug them in:

$$b= \binom{4}{0} x^4 \ y^0 + \binom{4}{1} x^3 y^1 + \binom{4}{2} x^2 y^2 + \binom{4}{3} x^1 y^3 \binom{4}{4} x^0 y^4$$

which is another way to say $(x+y)^4$

Side note: On tests, quizzes, etc. Professor Hochberg will tell Universe of Discourse.

$$(s + t)^n = \sum_{\substack{a+b=n \\ a,b\in N}} \binom{n}{b} s^a t^b$$

one way to simplify this would be to simplify n choose b

$$\binom{n}{b} = \frac{n!}{b!(n-b)!} = \frac{n!}{b!a!}$$

There are many ways to write this and this was Professor Hochberg's personal favorite.

which would simplify down to $\displaystyle\sum_{\substack{a+b=n \\ a,b\in N}} \frac{n!}{a!b!} s^a t^b$

With this proven, we can also go and use it with trinomials

Trinomial Theorem

$$(s + t + u)^n = \sum_{\substack{a+b+c=n \\ a,b,c\in N}} \frac{n!}{a!b!c!} s^a t^b u^c$$

and with 4 terms

$$(s + t + u + v)^n = \sum_{\substack{a+b+c+d=n \\ a,b,c,d\in N}} \frac{n!}{a!b!c!d!} s^a t^b u^c v^d$$

There is a shorthand for such factorial expressions

$$\frac{n!}{a!b!c!d!} = \binom{n}{a,b,c,d}$$ as long as they add up to n.

This is also called the multinomial coefficient

Note : we have already done this in a previous homework. The problem was similar to a store owner delegating positions to 10 different workers.

During the last class period, we expanded $(a + b + c)^4$

This would be similar to saying that we have 3 natural numbers that add to 4

Ways to arrange the 3 natural numbers

400	220
040	202
004	022
310	211
301	121
130	112
013	
103	

There turns out to be 15 ways to arrange these numbers

For each of these 15 cases, there would be a term with some coefficient in front of them

Note: On an upcoming assignment an example of two questions would be

1. Expand $(a + b + c)^4$
2. An if question concerning expanding $(a + b + c)^{20}$

We then took a quiz and went over the answers.

Question 1 was looked at and discussed. The question was to build Pascals triangle to the 8^{th} row.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 25 8 1

Then question 2 asked to circle a certain number according to its choose location

For example $\binom{4}{2} = 6$

You go to the row starting with "1 4" where the 1 is in the 0 position and the 4 is in the 1 position. You count to the 3 position and that gives you the corresponding number.

Finally, question 3 asked to take row twenty and compute alternatively the sum and difference of the numbers.

For example $\binom{20}{0} - \binom{20}{1} + \binom{20}{2} - \binom{20}{3} \,.\,.\,.\,.\,.\,.\,.\,.\,.\,.\, \binom{20}{19} + \binom{20}{20}$

The answer was 0 and Professor Hochberg told us to never draw out the 20^{th} row, but to look at the smaller rows and see the pattern.

Then he showed us the "Slickest Proof in the World", which is

Proof

$(1 - 1)^{20} = 0 = \sum_{i=0}^{20} \binom{20}{i} 1^{20-i} (-1)^i$