Discrete Mathematics --- Day 31 --- November 8, 2004

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Binomial Theorem:

For a natural number n:

$$(a+b)^n = \sum_{k=0}^{n} (\text{n choose k})\, a^{n-k} b^k$$

--Sigma notation means plug in various values of k from 0 to n and add up the terms.

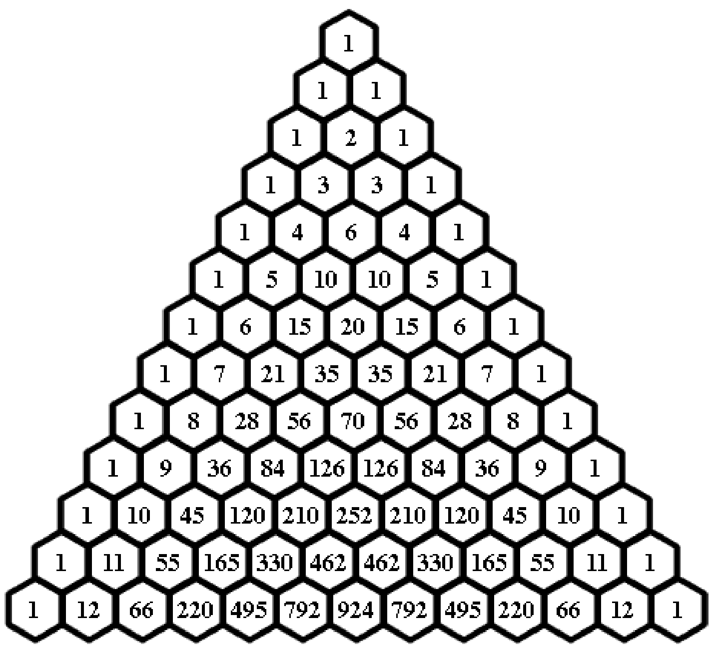
Eg: $(a+b)^3 = (a+b)(a+b)(a+b)$

$= (a+b)(a^2+2ab+b^2)$

$= (a^3+2a^2b+ab^2+a^2b+2ab^2+b^2)$

$= a^3+3a^2b+3ab^2+b^3$

A good way to show this is by using Pascal's Triangle:



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Eg:

1

$a + b$

$a^2 + 2ab + b^2$

$a^3 + 3a^2b + 3ab^2 + b^3$

....

This shows how Pascal's Triangle is relevant here.

Combinational Proof:

Consider $(a+b)^n = (a+b)(a+b)(a+b)(a+b)(a+b)...(a+b)(a+b)$
n Factors

Here we show it in an "anagram" type of way:

Consider multiplying 3 binomials together which are not all the same:
(a+b)(c+d)(e+f)
=(ac+ad+bc+bd)(e+f)
=ace+ade+be+bde+acf+adf+bcf+bdf
We see that the result is the sum of all ways to make a product by taking one term from each factor

So, consider:

$(a+b)^6 = (a+b)(a+b)(a+b)(a+b)(a+b)(a+b)$

Let's multiply this out "all at once." The product will consist of terms of the form $const \times a^i b^{6-i}$, where the constant is obtained by grouping together all the terms which have *i* "a" factors and 6-*i* "b" factors. That constant is just the number of anagrams of a word with *i* "a"s and 6-*i* "b"s.
To find out what row we can find our answer on in Pascal' s Triangle we find out how many "anagrams" this would have.
Eg:

$$\frac{6!}{2!4!} = 15$$

Therefore, row 6 of Pascal' s Triangle would give us our answer which is:

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$