Discrete Mathematics Notes – Day 28 – November 3, 2004

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Handout #6 _ Practice with Counting

1. How many 7-digit phone numbers are there that do not start with the digit "1"?

Obviously you can not start a phone number with the digit "0" either, so the phone number can now only begin with any digit 2-9.

There are 7 slots in a phone number:

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Each digit in the phone number can be between 0-9, except the first digit which can only be between 2-9. So there are 10 possibilities for each number, except the first, which only has 8 possibilities.

8 10 10 10 10 10 10

Which is also represented as $8 \times 10^6 = 8{,}000{,}000$ 7-digit phone numbers that do not start with the digit "1" or "0".

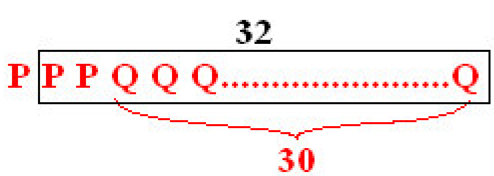
2. In a pretty good round of golf, Annika made 12 pars and 6 birdies. In how many ways could this have happened?

$$\frac{18!}{12!*6!}$$

which equals 18,564 different ways this could have happened.

3. 30 votes were cast in an election between Xavier, Yolanda and Zoe. How many outcomes were possible?

This is a "Quarters into Pockets" problem. There are 30 votes and 3 candidates. But one "P" must be the first symbol in the anagram.



So the answer would be 32!/(30!x2!) = 496.

If every candidate had to receive at least 1 vote, then you must reserve 3 quarters to give every candidate a vote. That leaves 27 Q's and 2 P's.

$$\frac{29!}{27!*2!}$$ **which equals 406 outcomes to the election.**

4. How many ways are there to toss 4 dice to obtain a sum of 9, if we count 1-2-3-3 as *different* from 1-3-2-3?

This is another "Quarters into Pockets" problem. There are 4 P's and 9 Q's, and none of the Pockets can be empty.

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So you must reserve 4 quarters so that each pocket has at least 1 quarter.

$$\frac{8!}{3!*5!}$$ **which equals 56 different ways to toss 4 dice to obtain a sum of 9.**

5. How many ways are there to toss 4 dice to obtain a sum of 9, if we count 1-2-3-3 as *the same* as 1-3-2-3?

For this problem, you must make a systematic list to obtain the number of ways: We don't have a slick counting method to do it.

6 1 1 1
5 2 1 1
4 3 1 1
4 2 2 1
3 3 2 1
3 2 2 2

There are 6 ways.