Discrete Mathematics --- October 29, 2004 --- Day 27

Class was started with a problem related to problem #9 on the homework.

The problem was posed of how to put 30 quarters into 6 Pockets.

Three combinations of 3 pockets will look like this:

28+1+1=30
27+2+1=30
27+1+2=30
etc…

At this point, a trick was shown to help with this kind of problem:

Using 3 Pockets and 5 Quarters

Pocket=P   Quarter=Q

Pocket 1	Pocket 2	Pocket 3	Symbolic Representation
2	1	2	PQQPQPQQ
0	2	3	PPQQPQQQ
5	0	0	PQQQQQPP
2	0	3	PQQPPQQQ
3	2	0	PQQQPQQP
0	5	0	PPQQQQQP

Since quarters can only go into pockets, an anagram of QQQQQPPP and such Starting with a Q cannot be used.

This can be interpreted as an anagram problem. Let's look at it that way.
So how many anagrams are possible of "PPPQQQQQ" which start with P?

Using past notes, it can be seen as this:

$$\frac{7!}{2!*5!}$$

Because we are finding anagrams of "PPQQQQQ," as the first "P" is fixed in its location.

How many anagrams are possible where no pockets are empty?

For each pocket, put one of the quarters into a reserve, and then figure out the anagram problem using the remaining quarters.

First Pocket-P   PPQQ   QQQ-Reserve
Anagram

The equation looks like this:

$$\frac{4!}{2!*2!} = 6$$

Back to problem #9 on the homework:

10 cookies and 4 children

Children-P Cookies-C

Anagram: PPPPCCCCCCCCCC

The reserve is 4 cookies, since we have 4 children.

Has to start with a pocket

Equation:

$$\frac{9!}{3!*6!} = 84$$

What if 25 pieces of paper were handed out to 20 students and each student got at least one?

Forces the first letter of the anagram to be S, where S is students and P is paper. Let us put 20 pieces of paper into reserve, so that we are finding the number of anagrams of a word with 20 S's and 5 P's, which start with 'S."

This makes the equation:

$$\frac{24!}{19!*5!} = 42{,}504$$

Another problem we looked at was the box problem. Move from top left to bottom right, only going down and right along the lines of the grid.

There are six moves right and four moves down. This can be put into an anagram of:

DDDDRRRRRR

This can also be solved as an anagram.

$$\frac{10!}{4!*6!}$$

Review of anagram notes:

If there is a required starting point, then it takes away that slot.

If each slot of one type must have another type following it, then subtract that many slots from the second type.

The student may wish to see the online notes which the teacher prepared for this topic as well.