Discrete Mathematics --- Day 26 --- October 27, 2004

In class today we recalled that a set with n elements has $\frac{n!}{k!\cdot(n-k)!}$ subsets of size k. $\frac{n!}{k!\cdot(n-k)!}$ is the same as $\binom{n}{k}$. Also, in class today we went over a few homework problems. One of the problems that we went over was problem number 30 on page 326. There was two parts to this problem.

Seven women and nine men are on the faculty in the mathematics department at a school. The first part wanted to know how many ways were there to select a committee of five members of the department if at least one woman must be on the committee. In order to solve this part of the problem we had to find out how many ways are there to select a committee of five members of the department of any sort. There were 16 members in all on the faculty in the mathematics department at the school. Five of the members had to be on the committee. In order to find out how many ways there are to select a committee of five members of any sort we would have to use $\left(\frac{16!}{5!\cdot 11!}\right)$. This is the same as $\binom{16}{5}$. When solving this we can cancel out some of the numbers in the numerator with all the numbers in the denominator. The number we got for the number of ways there are to select a committee of five members of any sort was 4,368 different ways.

Then we wanted to count the number of committees which had at least one woman on them. Our approach was to build a committee by placing a woman in the first position (7 ways to do this) and then select the rest of the committee from the remaining 15 faculty members ($\frac{15!}{4!\cdot 11!} = 1365$ways) for a total of $7 \times 1365 = 9555$ ways. Something that was wrong with this answer was that the number of committees with one woman or more was counted more than once. So we knew something was wrong.

When you count, count everything and don't count anything more than once.

Our next approach had us count the number of committees which have a woman on them by counting the number of committees which had no women on them (i.e., were all men) and subtracting that from the number found above of all committees.

To find the number of ways there are to select a committee of five members of the department without any women on the committee, all of the members on the committee would have to be men. There are nine men on the faculty in the mathematics department. There are $\left(\frac{9!}{5!\cdot 4!}\right)$, which is the same as $\binom{9}{5}$, ways to select a committee of five members of the department without any women on the committee.

The number we got for the number of ways there are to select a committee of five members without any women on the committee is 126 different ways. In order to find out the number of ways there are to select a committee of five members of the department with at least one woman on the committee we have to subtract the number of committees without women from the number of committees any sort. $|A| = |U| - |\bar{A}|$. A is the number of committees with at least one woman. U is the number of committees of any sort. The number we got for the numbers of ways there are to select a commit of five members with at least one woman on the committee is 4,242 different ways.

The second part of the problem wanted to know how many ways there are to select a committee of five members of the department if at least one woman and at least one man must be on the committee. We have already found the number of ways there are to select a committee of five members of any sort and the number of ways there are to select of committee of five members with at least one woman on the committee. Now we have to find the number of committees of five members without any men on the committee. We would have to use $\left(\frac{7!}{5!\cdot 2!}\right)$, which is the same as $\binom{7}{5}$ to find the number of committees of five members without any men on the committee. When solving this we can factor out some of the numbers. The number I got for the number of ways to select a committee without any men on the committee are 21 different ways. In order to find the number of ways there are to select a committee of five members of the department if at least one woman and at least one man must be on the committee we have to subtract the number of committees without women and the number of committees without men from the number of committees of any sort. $|A| = |U| - |Y| - |X|$. A is the number of committees with at least one woman and one man. Y is the number of committees without any women. X is the number of committees without any men. The number I got for the number of ways there are to select a committee of five members of the department if at least one woman and at least one man must be on the committee is 4,221 different ways.

Another problem that we went over in class was problem number 24 on page 325. This problem wanted to know how many ways were there for ten women and six men to stand in a line so that no two men stood next to each other. The hint that was given was to first position the women and then consider the possible positions for the men. If I position the women first, there are 11 possible positions for the men to stand. There are 10! Ways to arrange the 10 women. Then there are 11 ways to select where the first man goes. For the second man there are only 10 ways to select where he goes, because he can't stand in the position where the first man went. And so on for the next 4 men. So the total number of ways to line up these men and women is $10!\times11\times10\times9\times8\times7\times6$, which is the same as $\left(\frac{10!\cdot 11!}{5!}\right)$. When solving this we can factor out some of the numbers. The answer I got for the number of ways there are for ten women and six men to stand in a line so that no two men stood next to each other was 1,207,084,032,000 different ways.

Near the end of class we talked about the number of ways that we could place 5 quarters into two pants pockets and a shirt pocket without any of the pockets being empty. Here is a list of the ways, systematically listed such that we try to put as many quarters as possible in the earlier pockets before putting them in the later pockets. So altogether there are 6 ways.

Pocket 1	Pocket 2	Pocket 3
3	1	1
2	2	1
2	1	2
1	3	1
1	2	2
1	1	3