# Discrete Mathematics – Day 25 – October 25, 2004

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We started the class by getting questions from the homework.

p.310 #14

1 _ _ _ _ _ _ _ _ _ 1 Find # of n-length bit strings beginning and ending with 1
total number of spaces = $n$
so, the total number of possible bit strings equals $2^{n-2}$

Easy: 4 letter strings
_ _ _ _ total # of possible strings is $26^4$

4 letter strings having no "x"
_ _ _ _ total # of possible strings is $25^4$

4 letter strings which do have an "x" in them
= ( # of all strings) – ( # w/o "x")

Sometimes counting |A| is hard.
(where A is some set),

But finding $|\overline{A}|$ is easier.

$|\overline{A}| = |U| - |A|$

(and)

$|A| = |U| - |\overline{A}|$

pg. 312 #18a)

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n → | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | . . . |
| # of numbers ≤ n which are divisible by 7 → | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | |

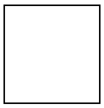
# of positive integers ≤ n which are divisible by 7 is $\lfloor n / 7 \rfloor$

floor function $\lfloor x \rfloor$ = x rounded down to the nearest integer

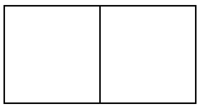
Ceiling function $\lceil x \rceil$ = x rounded up to the nearest integer

**Handout #1 -- Walking on some small grids**

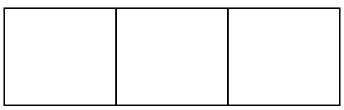
How many ways are there to walk from the top left to the bottom right corner in each of theses grids.



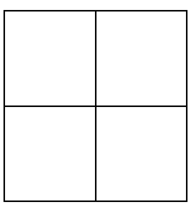
There are 2 ways to walk this grid
RD
DR



There are 3 ways to walk this grid
RDR
DRR
RDR



There are 4 ways to walk this grid
RRRD
RRDR
RDRR
DRRR



There are 6 ways to walk this grid
RRDD
RDRD
RDDR
DRRD
DRDR
DDRR

Then we figured out that all of the possibilities of the four square grid were all anagrams of “RRDD”. So for every way to walk there is an anagram for “RRDD”.

<u>Bijection</u>
ways to walk ←→ Anagrams of “RRDD”
1-1
correspondence

$$\# \text{ Anagrams} = \frac{4!}{2! * 2!}$$

$$= \frac{4*3*2}{2*2} = 6$$ so, there are 6 ways to walk

Then we said what if we had "RRRDD"

$$\# \text{ Anagrams} = \frac{5!}{3! * 2!}$$

$$= \frac{5*4*3*2*1}{3*2*2} = 10$$ so, there are ten ways to walk

Anagrams of "RRDD" ← key to many counting problems

↑

# of ways to choose k objects from n

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This is another problem that we did in class

Assign six children to three bedrooms
A, B, C, D, E, F

| 1 | 2 | 3 |
|---|---|---|
| AB | CD | EF |
| CD | AB | EF |
| AC | BD | EF |

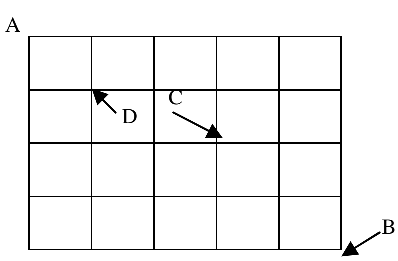
After trying to write the possibilities this way we decided that it would be more useful to write the possibilities this way:

| Children | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Bedrooms | 1 | 1 | 2 | 2 | 3 | 3 |

Now we can solve the problem like an anagram:

$$\# \text{ assignments is } \frac{6!}{2!*2!*2!} = 90$$ different assignments

Then we made this grid and used anagrams to figure out the # of different ways to get from one point to another.




4D - You have to go down 4 times
5R - You have to go right 5 times

1) A → B : $\frac{9!}{4!*5!}$ = 126

2) A → B through C

You need to find the # of ways to get from A → C

RRRDD = 10

And you need to find the # of ways to get from C → B

RRDD = 6

Then multiply 10 * 6 = 60

3) A → B without passing C

Take the total # of ways to get from A→B and subtract the # of ways to get from A→B through C.

$|A| = |U| - |\overline{A}|$

= 126 – 60

= 66

4) A→B without hitting D

= 126 – 2 * $\frac{7!}{3! * 4!}$

= 126 – 2 * 35

= 56

5) A→B without hitting either C or D

To do this problem we used this formula:

All ways – Ones that hit C – Ones that hit D + Those that hit both ( A→B→C→D )

= 126 – 60 – 56 + 2*3*6

= 126 – 60 – 56 + 36

= 46