Discrete Mathematics --- Day 21 --- October 13, 2004

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Gratuitous Proof by Induction:

Theorem: $\forall n \geq 2$, a set with n elements has $\frac{n(n-1)}{2}$ subsets of size 2.

Eg. - n = 4 {w,x,y,z}

Subsets of size 2: {w,x} , {y,z} , {x,y} , {w,y} , {x,z} , {w,z} $6 = \frac{4 \times 3}{2}$ ✔

Proof (by Induction):

Base Case: $n = 2$

Suppose set is {a,b}

It has only 1 subset of size 2, namely: itself and $\frac{2 \times 1}{2} = 1$ ✔

Induction Hypothesis:

Suppose it's true for some fixed value k. (←always *write this part*)

(*now elaborate*) That is, suppose any set of size k has $\frac{k(k-1)}{2}$ subsets of size 2.

Induction Step:

- Show its true for k+1.

Goal- Show a set with k+1 elements has $\frac{(k+1)(k+1-1)}{2}$ subsets of size 2. which is $\frac{(k+1)k}{2}$

- Consider a set X with k+1 elements

  Let a be one of the elements.

  X has 2 kinds of subsets with size 2: Those with a and those without a

  \# of subsets with a = k because if the first element is a, then there are k choices for what the second element of the subset should be.

  That is, the set should look like: {a, ≠a k(choices)}

  \# of subsets w/o a = $\frac{k(k-1)}{2}$ from I.H. because the set must look like:

  {≠a , ≠a }, and the two elements must be different., that is, this must be a Subset of a set of k elements

  Altogether, X has $\frac{k(k-1)}{2}$ + k subsets of size 2. (combining above choices)

(Solve) $\frac{k(k-1)}{2} + k = \frac{k(k-1)}{2} + \frac{2k}{2} = \frac{k(k-1+2)}{2} = \frac{k(k+1)}{2}$ ✔ Same as above!

^Get a common denominator

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*We took a group quiz

Counting

The # of ways to arrange (order) n distinct objects is $n! = 1 \times 2 \times 3 \ldots n$
(called n factorial)

*Another way to obtain that result:

Multiplication rule for counting

Given a task with k parts, each part of which must be completed in order to complete the task, If the parts have n_1, i_2, …, n_k ways to be completed respectively, then the Total # of ways to complete the task is obtained by: $n_1 \times n_2 \times \ldots n_k$

Eg - How many kinds of steak sauce could you make if a sauce consists of a letter followed by a digit?

A0	B0 …	Z0	**26**(*letters in the alphabet*)
A1	↓	↓	x**10**(*all numbers*)
A2	..		**260** (*Total # of ways*)
A3			
A4			
A5			
A6			
A7			
A8			
A9	B9	Z9	

***Powerful Trick/Advice**

- Turn questions of a "How many" variety into questions about "How many ways to comlete some task."

Rephrase above example = Select a letter and a digit to make a sauce.
$k = 2$
$n1 = 26$
$n2 = 10$
26 ways(letters) x 10 ways(digits) = 260 (multiply them!)

# of ways to order n distinct objects

Task: n , $(n-1)$, $(n-2)$, $(n-3)$, … , 2 , 1

1. Select 1st obj. -- (n) *corresponds to above*
2. Select 2nd obj. – $(n-1)$
3. Select 3rd obj. - $(n-2)$

…

n-1. Select the $(n-1)^{st}$ obj - (2)

n. Select the n^{th} obj. – (1)

Total # of ways = $n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 2 \times 1 = \boldsymbol{n!}$