Discrete Mathematics – Day 20 – October 11th, 2004

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Today, we had our tests from the previous Friday returned to us. There will be a redo exam this Friday, October 15th, 2004. Solutions to the exam will be posted on the website as well as an extra review session.

One major thing about tests: For test taking purposes, you need to do more than just build truth tables or membership tables. You also need to say why they prove what you are trying to prove. For example, you build the tables and circle the two equal columns, but you need to say what circling the columns means, such as "These two columns are equal."

Anyway, this was basically all about the test. For class, we covered another simple mathematical induction:

<u>Question</u>: How many ways are there to arrange (order) n distinct objects?

For this, we used a table to show the number of ways to arrange n objects.

n	**Number of ways**
0	0
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
n	$n \times$(number of ways for n-1)

We made this table by using letters and arranging them in all possible ways:

For 3 objects: ABC BAC CAB
ACB BCA CBA

Therefore, there are 6 total ways to arrange 3 objects.

-Whenever you are counting the number of ways to do something **Listing Systematically** is a very powerful technique. In this case, we listed the objects alphabetically.

For 2 objects: AB BA – There is only 2 ways to arrange 2 objects.

For 4 objects:

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

Notice when there are 4 objects to be arranged, there will be 4 different possible starting objects. Then you basically just rearrange the final 3 objects, which has already been done when we arranged 3 objects. Therefore there are 6 ways to arrange the remaining 3 objects and there are 4 columns. So, 6*4=24 – the number of ways to arrange 4 objects. Using this and noticing the pattern, for n objects, there will be: $n\times$(number of ways for n-1). To get the number of ways to arrange 5 objects, you would take 5*24=120, and so on to complete the table.

Definition: $n!$ ("n factorial") = 1*2*3*4*…*(n-1)*n.
Adding an $n!$ column to the table we constructed will look like this:

n	**Number of ways**	***n*!**
0	0	0
1	1	1
2	2	2
3	6	6
4	24	24
5	120	120
6	720	720
7	5040	5040
n	$n\times$(number of ways for n-1)	$n!$

Theorem: The number of ways to arrange n distinct objects is $n!$ ($\forall\ n \geq 1$)

Proof (by induction):

Base Case: n = 1 There is just 1 way to arrange 1 object AND 1!=1

Induction Step: Assume this is true for some fixed value k. So there are $k!$ ways to arrange any k distinct objects. Now consider $k + 1$ objects. There are $k + 1$ possible choices for which object goes first. There are $k!$ ways to arrange the remaining objects. So the total number of possible arrangements is $(k+1)k!$ According to the definition of $n!$, $k!$ multiplied by (k+1) is the same as saying $(k+1)!$ Therefore $(k+1)k!=(k+1)!$

At this point, our allotted amount of time for class had expired.