Discrete Mathematics--Day 18—October 6th , 2004

New requirements for notes-

All note pages must have a header that states “Discrete Mathematics—(day number)—(fully written out date)
Ex. “Discrete Mathematics—Day 1—August 25th, 2004”
This header should be on each page of the notes.

All mathematical variables used in the notes should be put into italics, and for the special set symbols “Z, Q, and R” the symbols should be bold and capitalized.

To create superscripts, it is done by going to format→effects→check superscript. It is a good idea to type a word or so after what you want to set as a superscript, so that you do not continue to make everything you type superscript.

<u>Homework #4</u>

1. The question here was “how many 10- digit numbers are there which don’t use any digit more than once, where no odd digit ever follows an even digit, and where the ‘4’ and ‘5’ are next to each other?”

5	4

In the space on the left of the 5 in box 1 and on the right side of the 4 in box 2. In box 1 the odd numbers left can be arranged in 24 different ways, and the same can be said for the even numbers in box 2. Therefore, to figure out the amount of numbers, you must simply multiply 24 by 24, which equals 576. Another way of looking at this is that the number of combinations for the even and odd numbers is the factorial of 4, and the factorial of 4 times the factorial of 4 is also 576.

2. The second problem on the homework required you to figure out how many different combinations of pizza toppings can be made using 7 different pizza toppings. Each topping has 2 possibilities: It either is or is not on the pizza. To figure out how many different combinations there are, you must simply raise 2 to the 7th power, which will come out to be 128 different combinations of pizza.

3. This problem involves a slight change in the original “Standing Bosses” game. Instead of the rightmost player standing and becoming the boss, the person second from the far right became the boss. Beyond this change the game is essentially the same as the original. For a game of 4 players, it would look something like this:

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_ _ _ 0 ← the 0 represents the player that is always sitting due to how the new rules work. The three dashes represent the players that are actually in motion during the game as it plays out. Since the rightmost player is always sitting, the order that the other three players stand and sit down is the same as counting in binary. This is what the 4 player game would look like:

0000→0010→0100→0110→1000→1010→1100→1110

Another way of looking at this sequence is that the 3 players who are able to move follow the sequence of a normal "Standing Bosses" game. Therefore, the formula for *n* players is 2^{n-1} positions.

4. This is another variation of the "Standing Bosses" game, but this time it is the rightmost player who does not have someone standing to their immediate left, and instead of everyone to the right of the boss, everyone else standing sits down. Some examples of the game would play out like this:

1 player-
0→1

2 players-
00→01→10

3 players:
000→001→010→100→101

4 players:
0000→0001→0010→0100→0101→1000→1001→1010

After playing out the games with four players, there is a noticeable pattern in who is sitting and who is standing. For example, in the four player game, it can be observed that the pattern up to the "0101" step is a repeat of the game for three players, and the remaining steps are a repeat of the game for two players. From this is can be surmised that any game using these rules uses the pattern of the previous two number of players, with the higher number of players going first.

Using the pattern described above, the number of moves for up to 15 players can be easily created using only the amount of moves for one player and two players. This is a table containing those values:

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# of players	# of positions
1	2
2	3
3	5
4	8
5	13
6	21
7	34
8	55
9	89
10	144
11	233
12	377
13	610
14	987
15	1597

To determine the number of positions for *n* players is $((1+\sqrt{5})/2)n+1 - ((1-\sqrt{5})/2)n+1$

5. This question contained another variation on the "Standing Bosses" game, but again with a change to the rules. For this variation, the bosses is the person to the left of the leftmost sitting player, unless the leftmost sitting player is the leftmost player, in which case the boss is the rightmost player. Due to the way this question was worded, the position sequence ends up being an infinite loop of the rightmost player standing up and sitting down. The game will never end because no one else other than the rightmost player will ever be able to stand up.

6. This question involves the "Standing Bosses" game, but there is no defined start position or order in which players will sit or stand. The question is to prove that a game with *n* players will either continue indefinitely, or stop within 2^n steps. To start, it can be said that there are only 2^n positions possible with *n* players, because the standing players are a subset of all *n* players. If a game using these rules with five players goes on for 40 turns ($40>32=2^5$) then the game will go on infinitely because the steps are starting to repeat themselves. Let *I* equal some position and *j* as some position such that ($j>I$). The positions at *I* and *j* are the same according to the rules of the game, and therefore $j+1$ must be the same as $I+1$ due to conditions in the description. Let $j=I+p$, so that every *p* turns we are back to the position that was turn *i*.

<u>Proof by Induction</u>
Find and prove a formula for the sum of the first n even positive integers.
$2+4+6+\ldots+2n=n(n+1)$

base case: $n=1$ $2*1=1(2)$ $2=2$ √

Induction Step: suppose true for some value *k*, that means $2+4+6+\ldots=k(k+1)$

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Show true- $2+4+6+\ldots+2k+2(k+1)=(k+1)(k+2)$

Consider $2+4+6+\ldots+2k+2(k+1)=$ (by IH) $k(k+1)+2(k+1)=(k+1)(k+2)$ √

Comment [TD1]:

By substituting in $(k+1)$ for just k in the original formula and solving out the resulting equation, it is possible to prove this formula for any sum of even numbers.