Discrete Mathematics --- Day 17 --- October 4, 2004

Reminders:

- 1^{st} Exam will be given Friday, Oct. 8.

HW Problem #6 on worksheet Hint:
Stage k 10010110100111
Stage k+1 is <u>determined</u> by Stage k 00001111000011

For ex. a game with 3 players
101 → 000 → ···

It will either stop or go on forever

More proofs by induction:

<u>Theorem</u>: $\forall n \geq 0,\ 3^0 + 3^1 + 3^2 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$

Proof (By induction on n):
There's two parts, the base case followed by an induction step.

Base Case:
When n = 0: $3^0 = \frac{3^{0+1} - 1}{2}$

Induction Step:
Suppose this is true for some fixed value k.

When, $3^0 + 3^1 + 3^2 + \cdots + 3^k = \frac{3^{k+1} - 1}{2}$

We want to show it is true for k + 1.

When we show that it's true for k we get to use it for k + 1 (thus, we get to use the induction hypothesis)

"Thought bubble" – We want to prove this

Show $3^0 + 3^1 + 3^2 + \cdots + 3^{k+1} = \frac{3^{k+2} - 1}{2}$

So, $3^0 + 3^1 + 3^2 + \cdots + 3^k + 3^{k+1}$

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Consider, $3^0 + 3^1 + 3^2 + \cdots + 3^k + 3^{k+1}$

By Induction Hypothesis, this equals $\frac{3^{k+1} - 1}{2} + 3^{k+1}$

Then get common denominator

$\frac{3^{k+1} - 1}{2} + 3^{k+1}$ so, $\frac{3^{k+1} - 1}{2} + \frac{2 \cdot 3^{k+1}}{2}$

$\frac{3 \cdot 3^{k+1} - 1}{2}$ same as, $\frac{3^1 \cdot 3^{k+1} - 1}{2}$

We add "powers" and get

$\frac{3^{k+2} - 1}{2}$ which is what we wanted, end of proof.

Theorem: $\forall n \geq 1,\ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \cdot \cdot \cdot + n(n + 1) = n(n + 1)(n + 2) / 3$

n	**Sum**
1	2
2	8
3	20
4	40
5	70

Proof (By Induction):

Base Case (n = 1)

$$1 \cdot 2 = \frac{1 \cdot 2 \cdot 3}{3}$$

Induction Step:

Suppose this is true for some fixed value of k:

$$1 \cdot 2 + 2 \cdot 3 + \cdot \cdot \cdot + k(k + 1) = \frac{k(k + 1)(k + 2)}{3}$$

Show it's true for k + 1

$$1 \cdot 2 + 2 \cdot 3 + \cdot \cdot \cdot + k(k + 1) + k(k + 1)(k + 2) = \frac{(k + 1)(k + 2)(k + 3)}{3}$$

Show it's true for k + 1 (cont.)

Consider $1 \cdot 2 + 2 \cdot 3 + \cdots + k(k+1) + (k+1)(k+2)$

(By Induction Hypothesis)

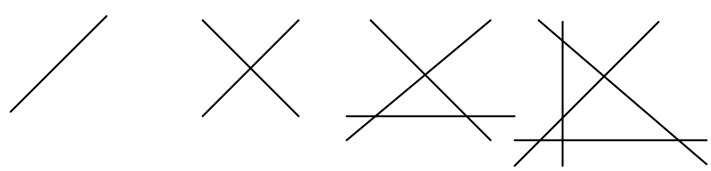
$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$ ← (factor out (k+1), (k+2))
← (common denominator)

$$= \frac{(k+1)(k+2)(k+3)}{3}$$ Which is what we wanted to show. End of Proof.

Theorem: $\forall n \geq 0$, the greatest # of points of intersection which n lines in the plane can create is

$$\frac{n(n-1)}{2}$$



n = 0	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6
0	0	1	3	6	10	15

#'s doubled:

0	0	2	6	12	20	30

Proof (By Induction):

Base case n = 0:

Induction Hypothesis:

Suppose it is true for some fixed value k. Then the greatest # of points of intersection among k lines is

$$\frac{k(k+1)}{2}$$

Show its true for k + 1, given k + 1 lines, there are at most

$\leq \frac{(k+1)(k)}{2}$ points of intersection

We'll continue, next class...