Discrete Math -- Day 14 -- September 27, 2004

In class today we began by going over the pattern of the standing bosses game. Which goes as follows:
1 2 1 3 1 2 1 4 1 2 1 3 1 2 1 5 1 2 1 3 1 2 1 4 1 2 1 3 1 6
As you can see in the pattern of the standing bosses game each underlined set repeats itself in each game. The non-underlined numbers are the number of participants of the game. We noted that this sequence gives the sequence of disk moves to solve the Towers of Hanoi puzzle, where even disks go left and odd disks go right.

Suppose on a test I asked you to list the 1st 10,000 terms of the "who's the boss" sequence.

Extra Credit

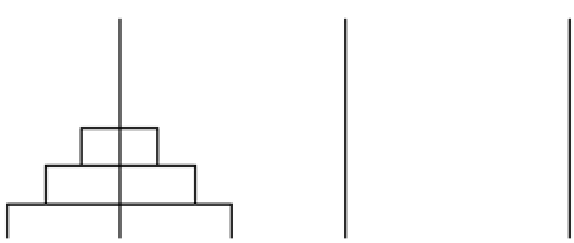
- What is the largest term?
- What is the 10,000th term?
- How many "1"s? **5000, because 1 appears as ever other number with the exception of the game number.**
- How many "5"s?
- Is there a "10"? If so, where is the first?

The Towers of Hanoi Puzzle

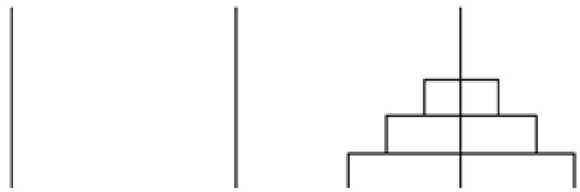
The Towers of Hanoi puzzle has 3 post and 64 disk that increase in size from the top to the bottom. Supposedly when the great monks move all of the disk from the first post to the last post the world is to end. This is to be done following these few rules:

1. Only one disk can be held in your hand at a time. The other disks must be on posts.
2. A disk can only be put down by placing it on one of the posts.
3. No disk can ever be placed on top of a smaller disk.

This is an example of what the puzzle looks like at the start.



And this what it should look like once you've completed it.



Theorem 2: 2 disks can't be moved in fewer than 3 moves.
Proof: To move the "2" we must move the "1" to another post then move the "2" then we replace the "1" atop the "2." That's 3 moves▪

Theorem 3: 3 disk can't be moved in fewer than 7 moves.
Proof: To move "3," you must first move the stack of 2 from post 1 to some other post. That is at least 3 moves. (See theorem 2.) Then move the "3" one move then move the stack of two atop the "3." That's a total of moves which are all required▪

Theorem 4: 4 disk can't be moved in fewer than 15 moves.
Proof: In order to move the "4," a stack of 3 must be moved & that takes 7 moves (as proved in Theorem 3). Then 1 move to move the "4" and then you must move the stack of 3 again which is 7 more moves. This shows that at least 15 moves must be required.

Ultimate Theorem: For all *n* in the set of natural numbers, *n* disk can't be solved in fewer than 2^n-1.

n # of disks	# of moves	optimal
1	1	Y
2	3	Y
3	7	Y
4	15	Y
5	31	Y
6	63	Y
7	127	Y
64	(2^64) -1	Y
n	(2^n)-1	Y

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