Discrete Mathematics – Day 11 – September 20, 2004

Set Equality

Two sets A and B are called "equal" if they have the same elements. That is if all elements of A are in B, and vice versa. We say $A=B$ if $A\subseteq B$ and $B\subseteq A$. When proving set equality ($A=B$), use either membership tables or show $A\subseteq B$ and $B\subseteq A$.

The elements of a set are not in any particular order within the set so {a,b,c}={a,c,b}. An element cannot be in a set "more than once." So {1,2,3,4,2,4,5} = {1,2,3,4,5}

The only time numbers are commonly repeated:
Let $S=\{1,x,x^2,x^3,x^4\}$
 when $x=2$ then $S=\{1,2,4,8,16\}$ $|x|=5$
 when $x=1$ $S=\{1\}$ and $|x|=1$
 when $x=-1$ $S=\{1,-1\}$ and $|x|=2$

The empty set- the set with no elements either written: { } or $\varnothing$
size: $|\varnothing|=|\{\}|=0$ is $\varnothing\subseteq\{1,2,3\}$? Yes.
The definition of $A\subseteq B$ is $\forall x(x\in A\rightarrow x\in B)$
$\forall x\,(X\in\varnothing\rightarrow x\in\{1,2,3\})$ This is vacuously true.
The empty set is a subset of every set.

Power set- if A is a set then we denote by P(A) the set of all subsets of A. For example:
 $A=\{1,2\}$ $P(A)=\{x|x\subseteq A\}=\{\varnothing,\{1\},\{2\},\{1,2\}\}$
the elements of the power set, are sets.
$A=\{1,2\}$ $A=\varnothing$, {1}
 ↑-----------↑ are not equal, one is a set one is an element.

Observation:
No finite set is ever a subset of its power set, except the empty set.

A	P(A)	elements of A
1	$\varnothing$	
2	{1}	
	{2}	
	{1,2}	

The power set of the empty set is the set containing the empty set:
$P(\varnothing)=\{\{\varnothing\}\}$ $\varnothing\neq\{\varnothing\}$ because, for example, $|\varnothing|=0$ and $|\{\varnothing\}|=1$
For all sets of finite or infinite,
$P(A)\neq A$
$|P(A)|=2^{|A|}$
example A = {a,b,c,d,e}, then $|P(A)|=2^5$ or 32

Set Operations

If A and B are sets, then we can define

$A \cup B=\{x \mid x \in A \vee x \in B\}$

$A \cap B=\{x \mid x \in A \wedge x \in B\}$

$A - B=\{x \mid x \in A \wedge x \notin B\}$

$A \oplus B=\{x \mid x \in A \oplus x \in B\}$ AKA the symetric difference

$\overline{A}=\{x \mid x \notin A\}$ (Including only those x's which are in the Universal Set **U**