

## Discrete Mathematics --- Day 10 --- September 17, 2004

Sets- A collection of objects.

- Sets are well defined- Know what elements are in the set and are not unambiguously.

Sets are written two ways

Explicitly-  $X = \{a, b, x, y, t\}$

This means set X contains five elements which are a, b, x, y, t

We use braces before each element and after the last element in a set.

Set  $Y = \{13, 31, 40, 4\}$

X is a set

eg.  $X + Y = ?$  This makes no sense because X and Y are sets and we haven't defined "+" for sets.

### Set Builder Notation

$$A = \{x \in X \mid p(x)\}$$

A = set's name

x = dummy variable (only in loop)

X = Universe of discourse for P's variable

p(x) = predicate

$\in$  = If A is a set and a is an element of the set A, we write it  
 $a \in A$  (English : a is an element of A)

eg.  $A = \{x \in X \mid p(x)\}$       born in May  
 $A = \{\text{Jason, Ryan}\}$   
(same set, with definitions written differently)

Jason and Ryan are the ones born in May, in our class.

If X is a set, the  $|X|$  denotes the number of elements of x, called the "size" or "cardinality" of X.

For example above     $|X| = 5$ ,  $|Y| = 4$ ,  $|A| = 2$

eg.

$X = \{\text{integers } x \mid x^2 = 4\} = \{-2, 2\}$  (to make  $x^2 = 4$ , -2 and 2 can be squared to equal 4 so therefore the size of the set would be:

$$|X| = 2$$

Some common sets:

$\mathbf{N} = \{0,1,2,3,\dots\}$  natural numbers

$\mathbf{Z} = \{\dots,-2,-1,0,1,2,\dots\}$  integers

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, a \neq 0\}$  rational numbers

$\mathbf{R} = \{\text{all numbers on the number line}\}$  real numbers

FYI:  $\mathbf{Z}$  is short for the German word Zahlen

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Is  $(\sqrt{2})^{\sqrt{2}}$  rational?

Suppose the answer is YES-

Then I have an irrational number to an irrational power being rational

Suppose the answer is NO-

The  $(\sqrt{2})^{\sqrt{2}}$  is irrational. So raise it to the  $\sqrt{2}$  power and it equals 2

So again we have irrational to an irrational power being rational.

Real answer: Nobody knows

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$\{x \in \mathbf{Z} \mid x/2 \in \mathbf{Z}\} = \text{set of even numbers (multiples of 2)}$   
(size of  $|\mathbf{X}| = \text{infinity}$  (bigger than any natural number))  
\*note: infinity is not a number  
\*note: the symbol for infinity “ $\infty$ ”

subsets: If A and B are sets, and each element of A is also an element of B, then we say A is a subset of B, written  $A \subseteq B$

also say

- B contains all of A's elements

eg.  $\{1,2,3\} \subseteq \{1,2,3,4,5\} = \text{True}$   
 $\{2,4,6\} \subseteq \{1,2,3,4,5\} = \text{False}$  because of 6

Suppose  $A \subseteq B$  and  $B \subseteq A$ . Then the sets are equal  $A=B$ .

We need to define what it means for two sets to be equal.

Definition: For sets A,B, we say  $A=B$  if

For all x ( $x(A \leftrightarrow x \in B)$ )

$\leftrightarrow =$  if and only if