Discrete Mathematics **Exam II - ANSWERS**

1. Construct a truth table for the expression (*p* ∨ *q*) w ¬*p*

p	*q*	*p*∨ *q*	¬*p*	(*p* ∨ *q*) w ~*p*
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

2. Negate each of the following compound propositions. Your answer should have no negated compound propositions.
   a. *p* 6 *q*
      i. (*p* ∨ ¬*q*)
   b. *p* ∨ *q*
      i. (¬*p* w ¬*q*)
   c. (*p* r *q*) 6 (*p* w *q*)
      i. (*p* r *q*) ∨ (¬*p* ∨ ¬*q*)
   d. (*p* r *q*) ∨ (*p* ø *q*)
      i. (*p* ø *q*) ∨ (*p* r *q*)

3. Write down the sentence that results when the following is negated twice:
   John met Mary and married her, but if she would have married James, then both Phil and Elaine would have been happier, and, if they would have lived to 100, would have had three children.
   Answer: The same sentence, because ¬¬*p* / *p*

4. Is the following a tautology: (*p* ∨ *q*) 6 (*p* 6 *q*)? *Yes*

5. Fill in the rest of this chart:

Name	Notation	In Words
Original Implication	*p* ÷ *q*	If you really love me, then you are willing to marry me
Inverse	¬ *p* ÷ ¬ *q*	If you don't really love me, then you are not willing to marry me
Converse	*q* ÷ *p*	If you are willing to marry me, then you really love me
Contrapositive	¬ *q* ÷ ¬ *p*	If you are not willing to marry me, then you don't really love me.

6. Of the four statements above, which pairs are logically equivalent to each other?
   The Original and the Contrapositive are logically equivalent to each other
   The inverse and the converse are logically equivalent to each other

7. Let B(*x*) be the predicate "Bill has visited *x*." Write each of the following in terms of B(*x*).
   a. Bill has visited Duluth
      B(Duluth)
   b. Bill has visited Shanghai, but not Mecca
      B(Duluth) ∨B(Mecca)
   c. Bill has visited everywhere
      œ*x*B(*x*)
   d. Bill has visited either Rome or Athens, but not both
      B(Rome) r B(Athens)

e. If Bill has been anywhere, then he’s been to Greenville
[›*x*B(*x*)] ÷ B(Greenville)

8. Let *P*, *Q* and *R* be predicates defined as follows: P(*x*): *x* has 3 elements; Q(*x*): *x* has an even number of elements; *R*(*x*, *y*): *x* and *y* have the same number of elements. Write each of the following statements in terms of *P*, *Q* and *R* and any quantifiers necessary.
   a. Every set has 3 elements
   œ*x* P(*x*)
   b. If a set has 3 elements, then it doesn’t have an even number of elements
   P(*x*) ÷ Q(*x*)
   c. If *x* has an even number of elements and *y* has an odd number of elements, then *x* and *y* have the same number of elements
   Q(*x*)∨¬Q(*y*) ÷ *R*(*x*, *y*)
   d. There is no set which has 3 elements
   œ*x* ¬P(*x*) is one possible answer. ¬›*xP*(*x*) is another, logically equivalent answer.

9. Given that *p* Y (*q* Y *r*) is false, what is the truth value of *r* Y (*p* Y *q*) ?
If the first expression is false, then p must be true, q must be true and r must be false. Plugging these values in, we see that the second expression must be true. So “True” is the answer.

10. Simplify each of these:
   a. $\binom{n}{0} = 1$
   b. $\binom{n}{1} = n$
   c. $\binom{n}{2} = n(n-1)/2$

11. Draw the first 8 rows of Pascal’s Triangle, and put a circle around the entry $\binom{7}{3}$
See the book, page 331, and circle the leftmost 35.

12. Expand $(r + s)^7$
Book, page 328

13. What is the coefficient of a^3b^4c in the expansion of $(2a + b + 5c)^8$?

$$2^3 \times 5 \times \frac{8!}{3! \times 4! \times 1!} = 11{,}200$$

14. Devise a logical expression which has the indicated truth table:
(¬*p* ∨ *q*)

p	*q*	???
T	*T*	*F*
T	*F*	*F*
F	*T*	*T*
F	*F*	*F*

15. Show that (*p* ∨ *q*) w ~*p* / (*p* 6 *q*). (*Hint: See Problem 1*)
The truth table for problem 1 is the same as that for (*p* 6 *q*).