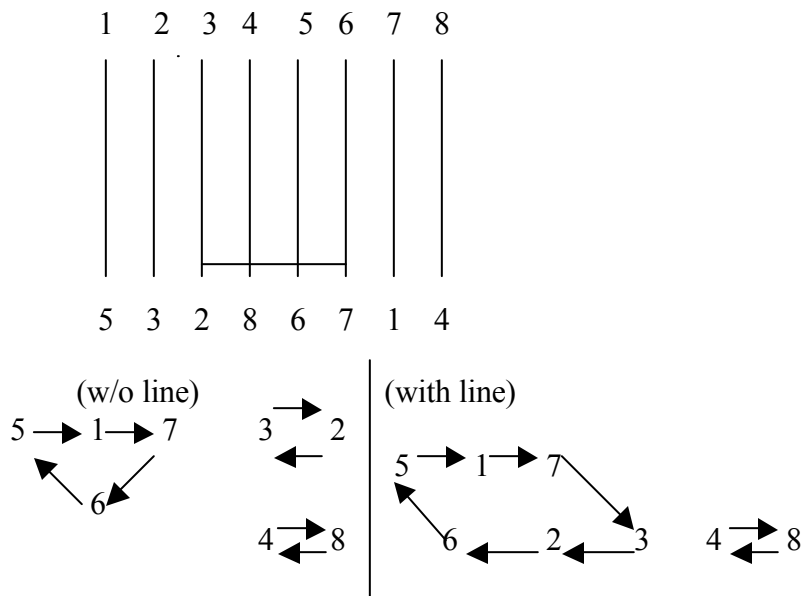


## Discrete Mathematics – Day 9 – 9/17/03

What happens to the permutation digraph if we add a line at the bottom of its web diagram?

- If the line connects two numbers in different cycles, then those 2 cycles join to become 1 cycle.



- If the line connects 2 numbers in the same cycle, then that one cycle becomes 2 cycles.

Note that we have covered all cases, so that we now know all that can happen when we add a single line to the bottom of a web diagram.

**Theorem:** A web diagram which generates an  $n$ -cycle must have (greater than or equal to)  $n-1$  lines.

**Proof:** Start with a web diagram with no lines. This generates the identity permutation which consists of  $n$  1-cycles. Now let's add lines 1 at a time to the bottom web diagram, and let  $k$  denote the number of cycles in the permutation. Initially,  $k=n$ . The addition of each line can reduce  $k$  by at least 1. (By connecting elements in diff. cycles) Our desired permutation is single  $n$ -cycle, where  $k=1$ . This will therefore require the addition of at least  $n-1$  lines. ■