Discrete Mathematics – Day 7 – 9/12/03

Home Work 2 Worksheet Passed out

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- http://www.cs.ecu.edu/~hochberg (course web site)
- 2427 – Discrete Mathematics
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$l'(B)$ = min # of lines needed if you connect only adjacent vertical lines
$t(B)$ = # of inversions in B

Theorem: $l'(B) = t(B)$

To prove this theorem, first observe that you can never fix more than one inversion with a horizontal line: This shows that $l'(B) >= t(B)$.

Now we show that when building a web diagram you can always find an adjacent pair out of order so a short horizontal line can fix that inversion. This means we never need more lines than inversions in the permutation, so that $l'(B) <= t(B)$.

But how can we show this?

Claim: In a permutation, if there is any inversion then there must also be an inversion of adjacent pair of elements.

Proof: Suppose A_i & A_{i+k} are inverted and k is atleast 1.
A_i -> $k - 1$ elements -> A_{i+k}

if k=1 then A_i and $A_{i+k} = A_{i+1}$ are adjacent
if $k >= 2$ then look at the pairs:

A_i, A_{i+1}
A_{i+1}, A_{i+2}
A_{i+2}, A_{i+3}
.....
A_{i+k-1}, A_{i+k}

the pairs above cannot all be in order or else:

$A_i < A_{i+1}$
$A_{i+1} < A_{i+2}$
.....
$A_{i+k-1} < A_{i+k}$

Which would imply by transitivity that $A_i < A_{i+k}$. But since A_i and A_{i+k} for an inversion we know that $A_i > A_{i+k}$.

We will be doing proofs all semester, at some point we will address the issue formally. Meanwhile, you can check out section 3.1 in the book for some pointers.

15 puzzle:

7	12	10	2
5	1	6	3
13	9	14	11
4	15	8	

Is this solvable? If it is a truly random permutation, then it has a 50/50 chance. Here is the permutation in standard notation:

7 12 10 2 5 1 6 3 13 9 14 11 4 15 8

Here is its cycle structure:
(1 6 7) (12 2 4 13 9 10 13 8 15 14 11) (5)

This means "1 goes to where 6 was, 6 goes to where 7 was, and 7 goes to where 1 was. In another cycle, 12 goes to where 2 was, 2 goes …"

We can now obtain the parity of the permutation by adding together the parities of the cycles. (Recall that even-cycles have odd parity and odd-cycles have even parity.)

3 cycle = even
11 cycle = even
1 cycle = even

Permutation = even