Discrete Mathematics – Day 6 – 9/10/03

Pep talk on Homework "Adjacent Transposition"
You may have found the HW to be difficult for two reasons: Notation and unfamiliar material.

For example, you are probably familiar with letting a variable stand for a number:

let x = Bill's age

But on this worksheet, we used Greek letters as variables, and the variable stood for a permutation, not just a number

p = permutation

Then there was the issue of using function notation. In this example:

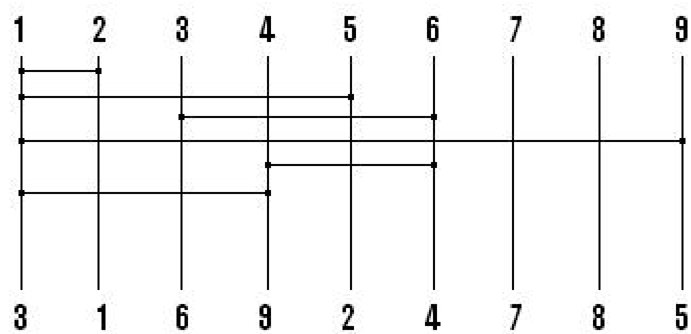
t(p) = # of inversion

the input to the function is a permutation, and the output is a number. And in this case:

l(p) = min # lines

the function outputs the minimum number of lines needed in a web diagram for the permutation. This is a function which, at this point, we don't know how to compute.

For example, consider the permutation 316924785. Here is a web diagram:



This web diagram has six lines, so we can conclude that

l(316924785) = 6

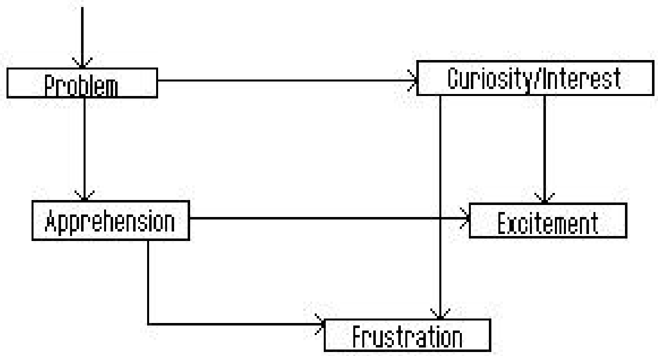
Why can we conclude this? Well, the diagram above uses 6 lines, so the *minimal* web diagram will use at most 6 lines. Since the web diagram above has 6 lines, we know the permutation is even, so the minimal web diagram requires either 6,4,2 or 0 lines. 0 and 2 lines are out of the question, because there are seven elements out of position, and 2 lines could put at most 4 elements into their correct positions.

On Homework 2, you will be invited to deduce a formula for the minimum number of lines needed

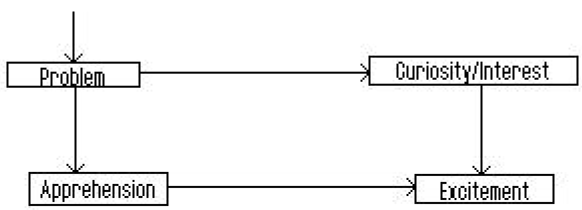
-Guideline for what to do when asked for a proof:

play for a while
discover why it's true
explain why it's true

A Little Bit about Emotions

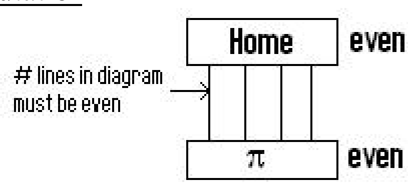


Emotions (cont.)



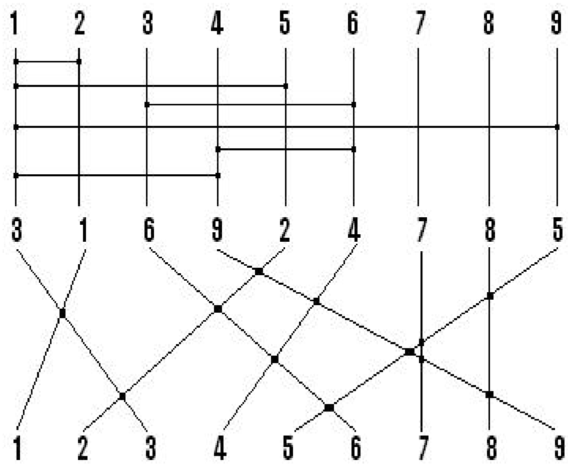
Question 1: Prove that t(p) and l(p) always have the same parity.

Suppose t(p) is even. Then p has an even number of inversions. In a web diagram the Home position has even parity (0 inversions) and every line that we add to the web diagram switches the parity of the resulting permutation



So the number of lines in any web diagram which generates π must be even. So l(π), in particular, must be even. If t(π) is odd then a similar argument shows we need an odd # lines, so l(π) is odd. ■

Question 2:



$l'(\pi)$ = smallest # of lines using only adjacent transpositions

$= \pi$ $l(\pi) = 4$ or 6

$t(\pi) = 12$

As we shall show below, $l'(\pi) = t(\pi)$.

Question 3: You need at most $t(\pi)$ lines because you can always find an adjacent pair that is out of order. (Proof to come.)

Question 4: You need at least $t(\pi)$ lines because each line fixes at most 1 inversion, and there are $t(\pi)$ inversions that need to be fixed.