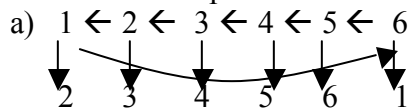


## Discrete Mathematics – Day 5 – 9/8/03

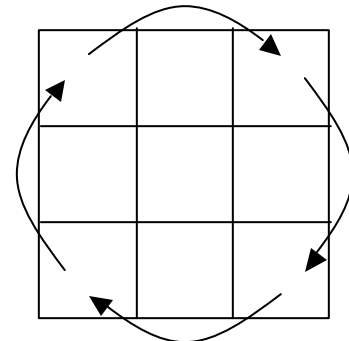
1) How to build a permutation in a cycle.



b) Is this permutation odd or even?

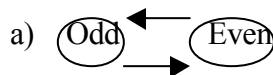
- i) Odd, because it is cycles an even number of elements
- ii) Ex: Rubik's Cube

On Rubik's Cube, every time we do a 90-degree turn of a face, we do a 4-cycle of the corners and a 4-cycle of the edges. (A 4-cycle of the corners is pictured to the right. The cycle of the edges would be similar.)



A solved Rubik's Cube is the "home" position, which has even parity. Every move changes the parity of the permutations of the edges and vertices.

2) Used 4-square-face to show parity change. Every 90-degree face turn changes the parity



i) even cycle numbers have odd parity

- 3) By swapping 1 pair, you prove that you can swap *any* pair of cubes. This is true because the pair we swapped could be any pair, and since there is no telling which two cubes you might choose, they must be able to swap in any position.
- 4) In the Rubik's Cube the corners (8 corners) can be in an odd or even permutation, same for the edges but, the permutations for corners and edges must always have the same parity.
  - a) Edges can't be even and corners be odd at the same time because they with each other.

With regard to a solved Rubik's Cube, there are two ideas: Each piece must be in its proper place, and in its proper orientation. So far we've discussed only placement of the corner and edge pieces. Now we will talk about orientation of these pieces. To do this, you consider the color tiles on the cube, of which there are 9 on each face.

5) Each 90-degree face turn moves 21 tiles.

- a) How do you find what parity each move makes the permutation of color tiles?
- b) Note that the four edge tiles on the turned face are cycled, as are the four corner tiles, and the 12 tiles along the sides of the top face that is being turned.
- c) Add up all the parities of each cycle (5 odds equal an odd permutation)

- 6) Theorem: You cannot flip a single edge on the Rubik's Cube.
- 7) Proof: every 90 degree face turn is an odd permutation of the 54 tiles. A solved cube with a single edge flipped is an odd permutation of the tiles. To solve the edge flip would require an odd number of 90 degree face turns. But as previously discussed, the edges and corners would both end up with odd permutations so it would not be in home position.
- 8) Can a cycle of six elements be done with 4 lines?
- a) No, it must be done with an odd number of lines.
  - b) We will talk on Wednesday about why 5 is the least number of lines that a 6-cycle can be done with.

