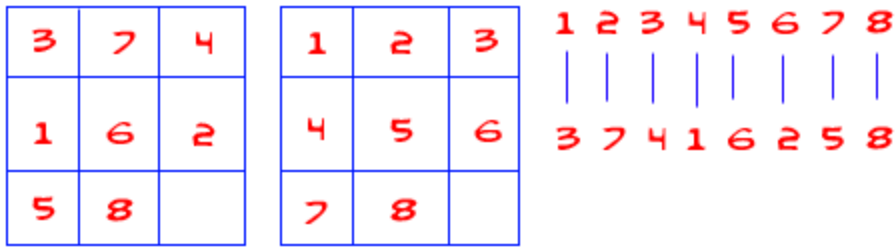
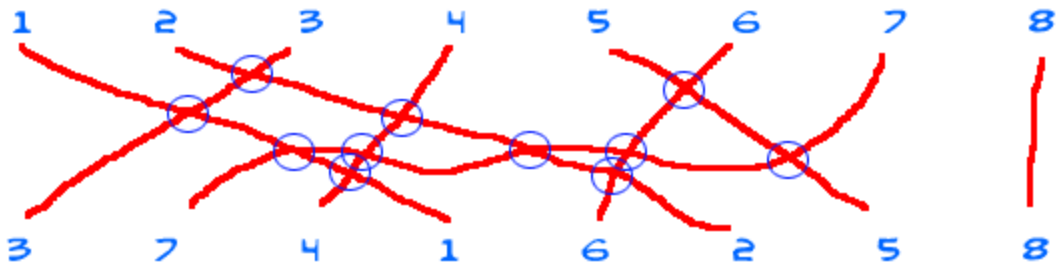


Discrete Mathematics – Day 4 – 9/5/03



Solved with at most 7 lines ($n-1$, where $n = 8$)



Why is an odd permutation unsolvable?

Thm: Odd permutations can't be solved. (Example found within the 15 puzzle, if you switch one and only one pair of numbers the puzzle became unsolvable...here is why)

Proof: Start with blank in bottom right corner (Oriented like the 15 puzzle started.)

Every move corresponds to a swap of some piece with the blank or marker piece.

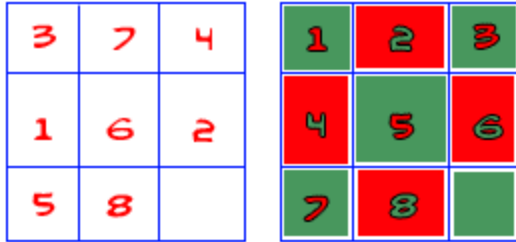
Let's write down the parity of board with every move.

<p>move: 3 7 4 1 6 2 5 8 b(lank) (odd parity)</p> <p>3 7 4 1 6 2 5 b 8 (even parity)</p> <p>*****</p> <p>*****</p> <p>1 2 3 4 5 6 7 8 b(lank) (even parity)</p>	<p>]</p> <p>]</p> <p>]</p> <p>]</p> <p>]</p>	<p>—————</p>	<p>***Note*** Each swap is by an odd amount.</p>
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Move:

Since each move swaps parity, and we started with an odd permutation, we must make an odd number of moves in order to restore the game to home position, because home is even.

Step 2. Color the checkerboard red & green with red on lower right. Every move switches the color that the blank is on. So, to return the blank to it's home position requires and even number of moves.



But a solution can't simultaneously require an even and odd number of moves. |

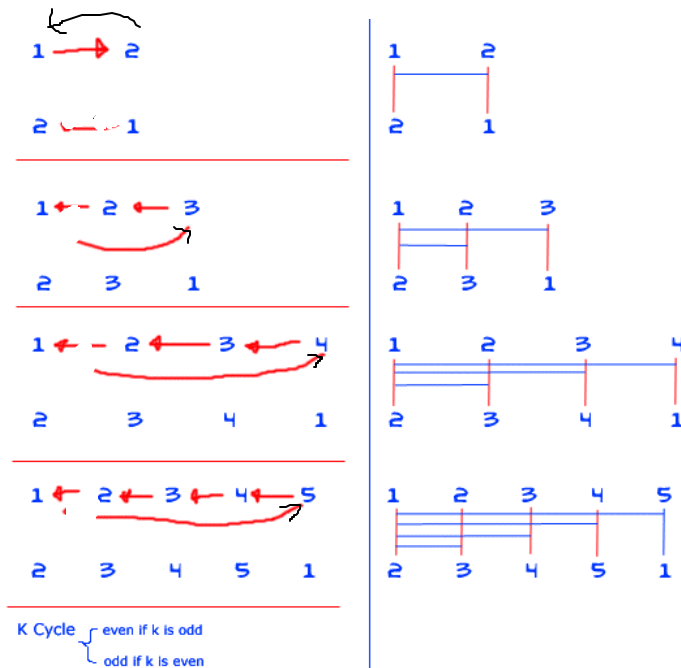
Side Note

In order to solve the 15 puzzle it's a series of even moves which allows you to return the game to it's original position 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, Blank

If the puzzle is odd, it would require an even number of moves to return the puzzle, and it's impossible to require both an even and odd number of moves at the same time, so the puzzle becomes unsolvable.

End Note

Now we turn our attention to cycles, and whether they are odd or even. The figure below shows some cycles and web diagrams for each of them.



Note that even cycles have odd parity and odd cycles have even parity.

For example, 5 is odd, and the web diagram which generates a 5-cycle has an even number of lines. So the 5-cycle is even.