

Discrete Mathematics -- Day 38 -- December 3, 2003

* “~” stand for “not”

Direct Proof

- Prove $p \rightarrow q$
- Do that by assuming p , and reasoning (math....) to show q must be true.

Indirect Proof

- Prove $p \rightarrow q$
- Do that by assuming q is false. Reason logically to show q must be false.

Proof by Contradiction

- Prove p
- Do that by assuming p is false, and deriving some absurdity, something clearly false.

Theorem: If n is an integer and n^2 is even, then n must be even.

Proof: Suppose not; n is an integer and n^2 is even, but n is odd. But this is absurd!! It would mean there is some odd integer which, when multiplied by itself, gives an even answer. Clearly false. (Odd * Odd != Even)

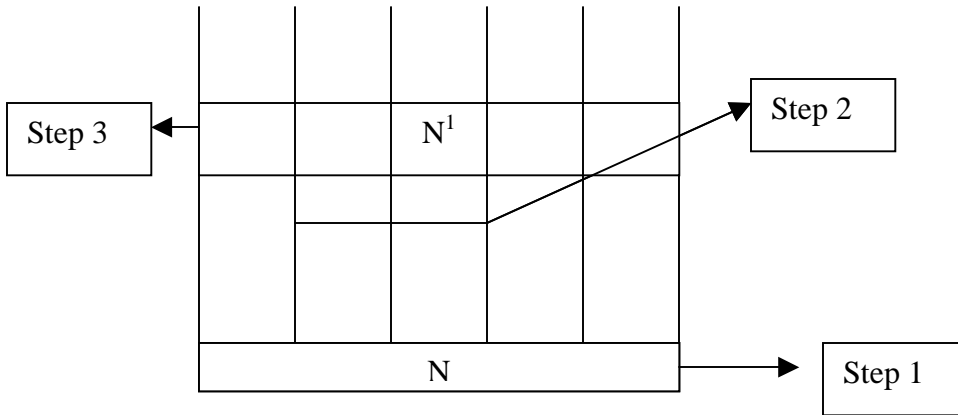
“Reductio ad absurdam”

- Sometimes a contradiction can be obtained by considering the smallest case where something impossible is alleged to happen.

Theorem: Every permutation has a web diagram.

Proof: Suppose not. Then there exists a permutation which has no web diagram. Each such permutation is not the identity permutation, for they have trivial web diagrams. Thus, every permutation with no web diagram has some elements out of place. Let N be a permutation that has the fewest # of elements out of place among all permutations with no web diagram

We will show N does in fact have a web diagram:



Step 1: Put N here.

Step 2: Add a single line at the bottom to fix at least one element N (put it in place)

Step 3: Call the resulting permutation N^1 .

N^1 has fewer elements out of order than N. Thus, by the manipulation of N, N^1 does have a web diagram. But that web diagram, together with that bottom line, is a web diagram for N. (Contradiction)

Cheesy Proof Technique

E.g. Prove that the following argument is valid

$p \rightarrow q \rightarrow r$

$\sim r$ _____

$\therefore \sim p \wedge \sim q$

Prove by truth table

- Cross out rows where the givens are false and show that the conclusion is true in all remaining rows.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$\sim r$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	T	T	F	F	F	F
T	T	F	T	F	T	F	F	F
T	F	T	F	T	F	F	T	F
T	F	F	F	T	T	F	T	F
F	T	T	T	T	F	T	F	F
F	T	F	T	F	T	T	F	F
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T