# Discrete Math – Day 35 – November 21, 2003

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Logic
Propositions
Predicates
Quantifiers
Compound Statements
*Rules of Inference

What Makes an Argument Valid?

- Pi is a Permutation of 9 elements
- Pi has 13 inversions
- Pi's minimum web diagram has atleast 7 lines
- Therefore Pi consists of 2 cycles

Rules of inference are the proofs used to prove this.

1. Odd number of inversions ↔ odd permutation ↔ Odd number of lines
2. Minimum number of line = n – the number of cycles
3. Minimun number of linkes ≤ n – 1

Pi has 9 elements → Pi's minimum web diagram has ≤ 8 lines (l(pi) ≤ 8)

$l(pi) \leq 8 \leftrightarrow (l(pi = 0) \vee (l(pi)=1) \vee (l(pi)=2) \vee (l(pi)=3) \vee \ldots \vee (l(pi)=8)$
$l(pi) \geq 7 \rightarrow \neg(l(pi = 0) \wedge \neg (l(pi)=1) \wedge \neg (l(pi)=2) \wedge \neg (l(pi)=3) \wedge \ldots \wedge \neg (l(pi)=6)$

Modus Ponens
p
$p \rightarrow q$
$\therefore q$

If the top is true then the bottom is true otherwise the whole thing is false.

Proof of Modus Ponens:

| p | q | p→q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Cross out bottom three rows as the premises are not true.
Check conclusion in remaining row.

Hochberg Ponens
p
¬q→r
p∧ ¬q
∴ r

p ∧ q ∧ r ∧ s ∧ t
∴ p
∴ q
∴ r
∴ ∴ s
∴ t

p ∨ q ∨ r ∨ s ∨ t
¬p
¬q
¬r
¬s
∴ t

p ∨ q ∨ r ∨ s ∨ t
¬p
¬q
¬r
∴ s ∨ t

p↔q ≡ (p→q) ∧ (q→p)

l(pi) ≤ 8 ↔ (l(pi = 0) ∨ (l(pi)=1) ∨ (l(pi)=2) ∨ (l(pi)=3) ∨ … ∨ (l(pi)=8)

r = l(pi) ≤ 8
s = (l(pi = 0) ∨ (l(pi)=1) ∨ (l(pi)=2) ∨ (l(pi)=3) ∨ … ∨ (l(pi)=8

p (given)
p→q
∴q
q→ each of it's conjunctive parts (simplification)

r (given)
(r↔s) → (r→s) (simplification)
∴ r→s ∴ s

$l(pi) \geq 7 \rightarrow \neg(l(pi = 0) \wedge \neg (l(pi)=1) \wedge \neg (l(pi)=2) \wedge \neg (l(pi)=3) \wedge \ldots \wedge \neg (l(pi)=6)$

$p = l(pi) \geq 7$
$q = \neg(l(pi = 0) \wedge \neg (l(pi)=1) \wedge \neg (l(pi)=2) \wedge \neg (l(pi)=3) \wedge \ldots \wedge \neg (l(pi)=6)$

$l(pi) \leq 8 \leftrightarrow (l(pi = 0) \vee (l(pi)=1) \vee (l(pi)=2) \vee (l(pi)=3) \vee \ldots \vee (l(pi)=8)$

$r = l(pi) \leq 8$
$s = (l(pi = 0) \vee (l(pi)=1) \vee (l(pi)=2) \vee (l(pi)=3) \vee \ldots \vee (l(pi)=8$

s consists of 9 disjunctive parts 7 of them are false by q so by disjunctive syllogism at least one of the remaining 2 must be true.

$\rightarrow (l(pi) = 7) \vee (l(pi)=8)$

$u = (l(pi) = 7)$
$w = (l(pi) = 8)$

pi has 13 inversions → pi is odd → l(pi) is odd → $l(pi) \neq 8 \leftrightarrow \neg(l(pi) = 8)$

$\neg(l(pi) = 8) = \neg w$

$u \vee w$
$\underline{\neg w}$
$\therefore u$

$\rightarrow l(pi) = 7$

Theorem
$l(pi) = n - c$
$7 = 9 - c$
$c = 2$