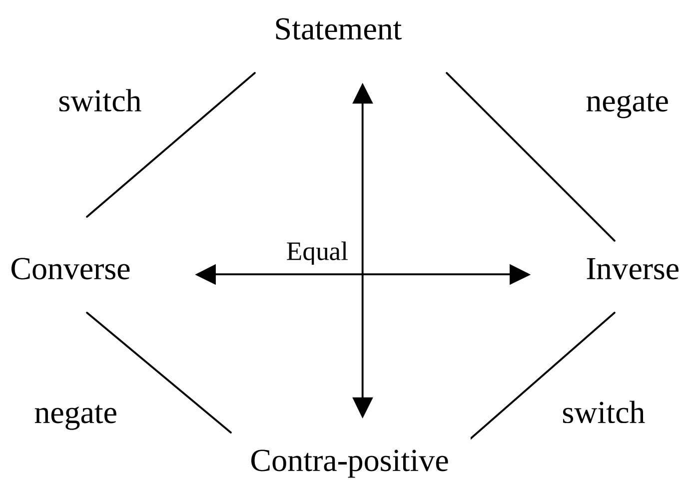
Notes for Friday November 14, 2003

Implications

- "if p, then q"
- "p implies q"
- "q if p"
- "p only if q"
- "q is necessary for p"
- "q whenever p"

You can make other implications from p → q

- p → q is a **statement**
- q → p is its **converse**
- ¬ p → ¬ q it its **inverse**
- ¬ q → ¬ p is its **contrapositive**



"The understood ∀" – Sometimes you'll see an implication with a variable in it:

- "if x is even, then x^2 is even."
- If 7 is even, then 49 is even."
    - False → False = True
- There is an "understood ∀" in front of the statement
    - "∀x(~~~)"
- if π has an even number of inversions, then a web-diagram for π must have an even number of lines
    - it is understood that the universe of discourse of π is a permutation
- What is the converse of the statement above?
    - If π's web diagram has an even number of lines, then π has an even number of inversions
- What is the inverse?
    - If π does not have an even number of inversions, then π's web diagram has an odd number of lines
- What is the contra positive?
    - π's web diagram has an odd number of lines, then π has and odd number of inversions

refer to the diagram above:

- notice that the statement and contra-positive is logically equivalent.
    - $p \rightarrow q \equiv \neg q \rightarrow \neg p$
- notice that the converse and inverse are logically equivalent
    - $q \rightarrow p \equiv \neg p \rightarrow \neg q$

Negation

- $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- $\neg(p \text{ v } q) \equiv \neg p \wedge \neg q$
- $\neg(p \oplus q) \equiv p \leftrightarrow q$
- $\neg(p \rightarrow q) \equiv p \wedge \neg q$
- $\neg(p \leftrightarrow q) \equiv p \oplus q$
- $\neg \neg p \equiv p$