Day 31

Logic Compound Propositions Nov 12, 2003

$p \wedge q$	True when P and Q are true
$p \vee q$	True when P or Q or P and Q are true
$p \oplus q$	True when P or Q is true only
$p \rightarrow q$	False when P is true but Q is false
$p \leftrightarrow q$	True when P and Q are both true or both false
$\neg p$	Opposite value of P

Truth Tables

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	$\neg P$
T	F
F	T

$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$

↳ logically equivalent regardless of what they are

Verify that two expressions are logically equivalent with truth tables

Claim $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Proof

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

Claim: $P \to (Q \to R) \equiv \neg P \vee \neg Q \vee R$

Proof:

P	Q	R	$P \to (Q \to R)$	$\neg P \vee \neg Q \vee R$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

$\vee$ is associative $(P \vee Q) \vee R \equiv P \vee (Q \vee R)$

$(P \wedge Q) \wedge R \equiv P \wedge (Q \wedge R)$

Parentheses unnecessary when using several disjunctions or conjunctions

$(P \vee Q) \wedge R \not\equiv P \vee (Q \wedge R)$