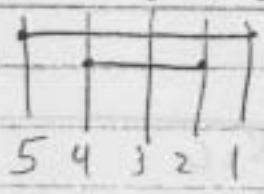
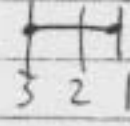
Day 3

Sept 3 2003

#2 1 2 3 4 5 Two lines

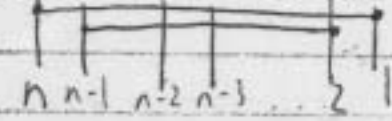


* 2. 1 2 3 one line



minimum

* 1 2 3 4 ... n the number of lines you need is $\frac{n}{2}$ rounded down



* $\frac{n-1}{2}$ if n is odd

$\frac{n}{2}$ if n is even

} for the number of lines

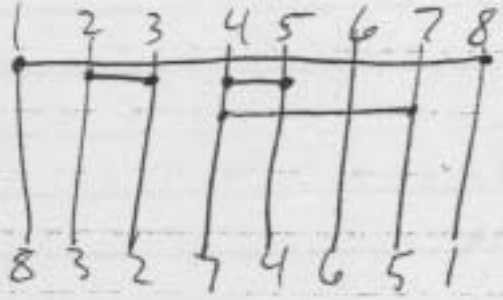
* in computers you would use int(n/2)

"$\frac{n}{2}$ rounded down" is $\lfloor \frac{n}{2} \rfloor$

* $\lfloor x \rfloor$ = x rounded down to nearest integer
floor function

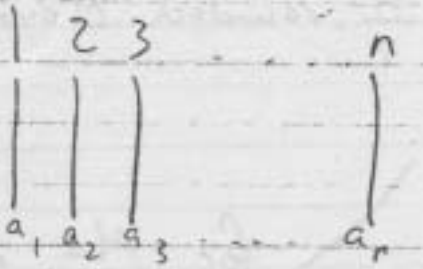
* $\lceil x \rceil$ = x rounded up ceiling function

...prove any permutation can be obtained



K=6
K is in the right position

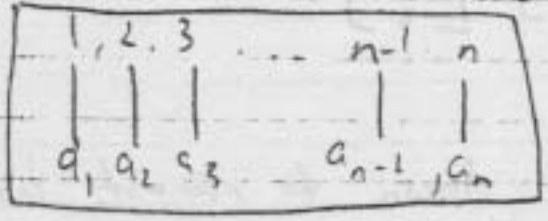
1 start with permutation



K are in correct position
I can add a line to increase K by one

ed * Theorem: We can obtain any given permutation via a web diagram.

ue Proof: Suppose there are n elements, and we wish to obtain the permutation $a_1 a_2 a_3 \ldots a_{n-1} a_n$ $\{1, 2, 3, \ldots n\}$
Start with the web diagram



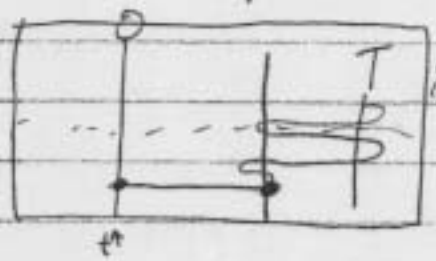
and let K denote the number of elements already in their correct positions

(continued)

(continued)

Let us add lines one at a time to the bottom of our web diagram as follows:

- Find an element, call it t, that is out of place and add a line to put it into its correct position.



T is not in correct position

T is move now

Note that this line does not remove the swapped element from a correct position because that element was heading for the position where t belonged and therefore was not in its correct position.

This increases K by at least one and possibly two if the swapped element also ends up in the right place.

Finally note that, when K reaches n, we have obtained our desired permutation. ∎

Corollary: Any permutation of n elements can be obtained from a web_diagram with at most $n-1$ horizontal lines. Last line goes up by 2.