Day 29 – Discrete Mathematics – November 7, 2003

Logic:

Proposition/Statement- An unambiguously true or false assertion (i.e. 2+2=4)

Predicates- Propositions with variables, not really a proposition because not unambiguously true or false (i.e. x+2=4)

Qquantifiers- Bind the values of predicates to turn them into propositions. There are two of these: For all($\forall$) says that every value in the universe of discourse could be inserted.
There exists($\exists$) says that there is at least one value in the universe of discourse that can be substituted for the variable.

Universe of Discourse- Those values, things, etc. that can be considered in a predicate. It must be defined, because otherwise it can lead to confusion.

Rules of Composition- Ways to build compound propositions.

Rules of Inference- Rules for establishing the truth of a proposition.

Examples:

Proposition: 1+1=2. This is unambiguously true, all values are explicitly stated

Predicate: x=3. It is unknown whether this is true or not because the variable has not yet been bound and there is no universe of discourse.

Quantifiers and Universes of Discourse:

p(x)$\equiv x$=3, for all integers. The predicate, p, now has a universe of discourse (all integers), and can now be bound by quantifiers.

$\exists x$p(x), reads "there exists a value of x such that x=3" this statement is now a proposition because it is unambiguously true. There is an integer that equals 3.

$\forall x$p(x), reads "for every possible value of x, x=3" this is also now a predicate because it is unambiguously false, not every integer is equal to 3.

More complex predicates:

How can we express "Every student got an A" using predicates? First we need to decide on what to express as a variable. Since the grade, A, is fixed that means that the student has to be a variable. That would give a predicate similar to the following: p(x)="x got an A". But this isn't enough, we also need to define a universe of discourse. Since we are talking about a class we can define the universe of discourse as the set of students in the class. All that is left is to bind the variable with a quantifier. Since the original sentence says “every student” we’ll use for all. This gives the following:

p(*x*)=”x got an A”
In the set of all students in the class
$\exists x$ p(*x*)

b(*x*)=”*x* got a B”
In the set of all students in the class
$\exists x$ b(*x*)
This translates to “Somebody in the class got a B”

Predicates cannot stand on their own, as in m(*x, y*)=”*x* likes *y*”, because the universe of discourse is not always clear. In this example *x* and *y* could be anything from people to numbers. In order to make it useful we have to define the universe of discourse. For example:

m(*x, y*)=”*x* likes *y*”
x is a student a ECU
y is a type of fish

Now m(x, y) can be used to make statements like “Rob likes tuna”:

m(Rob, tuna) (note, however that“Tuna likes Rob” $\neq$ m(tuna, Rob) because tuna is not a student and Rob is not a fish.)

To express “Erin likes all kinds of fish” you would write:

$\forall y$ m(Erin, *y*)

or, if you wanted to say “Somebody like salmon” you would write:

$\exists x$ m(*x*, salmon)