Discrete Mathematics — Days 20 and 21 — October 17 and 20

Expand $(r+s)^7$ $= r^7$ power of r decrease
Powers of s increase

Example. $r^7 + r^6s + r^5s^2 + r^4s^3 + r^3s^4 + r^2s^5 + rs^6 + s^7$

$1r^7 + 7r^6s + 21r^5s^2 + 35r^4s^3 + 35r^3s^4 + 21r^2s^5 + 7rs^6 + 1s^7$
$\binom{7}{7}$ or $\binom{7}{7}$ + $\binom{7}{6}$ or $\binom{7}{1}$ + $\binom{7}{5}$ or $\binom{7}{2}$ + $\binom{7}{4}$ or $\binom{7}{3}$ + $\binom{7}{3}$ or $\binom{7}{4}$ + $\binom{7}{2}$ or $\binom{7}{5}$ + $\binom{7}{1}$ or $\binom{7}{6}$ + $\binom{7}{7}$ or $\binom{7}{7}$
*note: You can also use pascals triangle

$(x + y)^n$ - to expand, take the row (n) of pascals triangle. Row 7 = 1-7-21-35-35-21-7-1

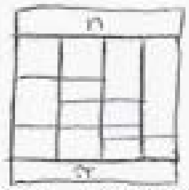
Question. I have 10 student's exam's and I want to assign 4 A's, 3 B's, 2C's, and 1 D to these exams.
How many ways can this be done ?

Answer 1. 10! / 4!*3!*2!*1! = 10*9*8*7*6*5*4*3*2*1 / 4*3*2*1*3*2*1*2*1*1 = 12,600

We can assign the exams to: Q R S T U V W X Y Z
And assign grades to them: A A A A B B B C C D <- every assignment of grades is an Anagram of "AAAABBBCCD"
So another way to answer is: 10 choose 4 * 6 choose 3 * 3 choose 2 * 1 choose 1 = 12600

Question 1. What is an inversion – 2 elements out of place

Question 2.



What is the parity of π – EVEN because it has 4 lines.

Question 3. Can $\pi = (12)(345)$ – NO, because the parity is odd, and the parity of the permutation is even.

Question 4. If $n = 9$ and the number of lines = 11, what is the minimum number of lines you will need to complete the permutation.
Answer – normally the answer would be n-1, so there would be < or = 8lines needed. But in this case, the parity is odd (we know this because there are 11 lines, and 11 is odd) so this means that the least number of lines will be 7 or less.

Expand $(x + y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y) = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
The sum of the exponents always stay balanced = 5

When multiplying polynomials every term in $(a + b + c)$ must be multiplied by $(d + e + f)$ when multiplying $(a+b+c)(d+e+f)$

How to multiply more than 2 polynomials:
$(a+b)(c+d)(e+f) = (ac+ad+bc+bd)(e+f) = (ace+ade+bce+bde+acf+adf+bcf+bdf)$
Just pick all the possible ways to pick one term from each bracket.
The final answer is the sum of all products formed by selecting one term from each factor

The difference between factors and terms. Terms are added and subtracted, whereas factors are multiplied.
Terms –> a + b
Factors –> (x+y)(x+y)

$\underline{1}x + \underline{5}x^4y + \underline{10}x^3y^2 + \ldots.$

The <u>number</u> of ways to select x and y ..... for 1x there are $\binom{5}{5}$ ways to select x. and for $\underline{5}x^4y$ there are $\binom{5}{4}$ or $\binom{5}{1}$ ways to select x and y

There is a 1 to 1 correspondence between ways to select 3x's and 2y's from the factors of $(x + y)^5$ and anagrams of the word "xxxyy"

A consequence of this – we know the coefficient in $x+y^n$ of x^ky^{n-k} is going to be $\binom{n}{k}$ or $\binom{n}{n-k}$

Binomial Theorem : $(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^{n-i} y^i = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1}y + \binom{n}{2} x^{n-2}y^2 + \ldots\ldots$
$\ldots\ldots\binom{n}{n} x^{n-n}y^n$