Discrete Mathematics --- Day 19 --- Wednesday, October 15

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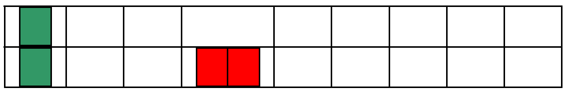
H.W. # 3 solved in class:

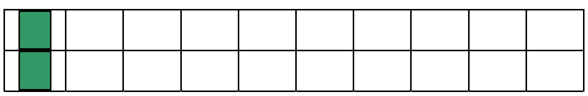
1) Prove the domino recursion.

D_n = # ways to dominize 2x*n* board.
$D_n = D_{n-1} + D_{n-2}$, for $n >= 3$.



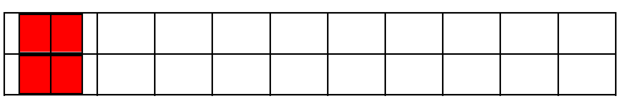
Green=Vertical domino,
Red= Horizontal domino.

Let's compute D_n. Consider the upper left square. It can be covered in two mutually exclusive ways: Vertically or Horizontally.
If we cover it vertically, then what remains is a 2 x (n – 1) board, which has D_{n-1} ways to be covered.



|____________ n – 1 _________________|

If we cover that square horizontally, then a horizontal tile must go below it, leaving a 2 x (n – 2) board, which can be tiled in D_{n-2} ways.



|____________ n- 2 ___________________|

So, altogether there are $D_{n-1} + D_{n-2}$ ways to cover the 2xn board, so

$$D_n= D_{n-1} + D_{n-2}.$$

∎

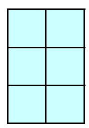
Note: Proofs often have the collection of math and paragraphs explaining it and not just mathematical expressions alone.

2) Dominize 6x20 board with 2x3 dominos.

There are different ways to find the answer.

1. To use tree diagram : difficult and lengthy.
2. To see the 1st square, and see if that square has Vertical or Horizontal domino.
   If Vertical, then it would have D_{n-2} ways.
   If Horizontal, then it has D_{n-3} ways.
   So, $D_n = D_{n-2} + D_{n-3}$.
3. To start finding the # ways to dominize smaller sizes and keep increasing them until a pattern is found:

Dn = Dn-2 + Dn-3.



Size	# ways to dominize V or H dominos
6x1	0
6x2	1
6x3	1
6x4	1
6x5	2
6x6	2
6x7	3
6x8	4
6x9	5
6x10	7
6x11	9
6x12	12
6x13	16
6x14	21
6x15	28
6x16	37
6x17	49
6x18	65
6x19	86
6x20	114

e.g. (D_{n-3}) 5
\+
(D_{n-2}) 7
12

So, the number of ways to dominize a 6x20 board is 114.

3) Find a formula for

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots\ldots\ldots\ldots\ldots\ldots+\binom{n}{n}$$

Let $S_n = \binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots\ldots\ldots\ldots+\binom{n}{n}$

$S_n = 2(S_{n-1})$.

n	Sum	
1	$\binom{1}{0}+\binom{1}{1} = 2$	$= 2^1$
2	$\binom{2}{0}+\binom{2}{1}+\binom{2}{2} = 4$	$= 2^2$
3	$\binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3} = 8$	$= 2^3$
4	16	$= 2^4$
…	…………………….	
…	…………………….	
…	…………………….	
n	……………………	$=2^n$

Why does $\binom{n}{0}+\binom{n}{1}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+\binom{n}{n} = 2^n$?

Combinatorially (By Counting):

$\binom{n}{0}$ = # ways to pick 0 elements out of n.

$\binom{n}{1}$ = # ways to pick 1 element out of n.

$\binom{n}{2}$ = # ways ………2 elements out of n.

…………………………………………….

…………………………………………….

$\binom{n}{n}$ = ………………n elements out of n.

4) Write a paragraph justifying it.

Each number in the sum counts the # of subsets of a particular size. So, the sum of all those numbers computes the total # of ways to pick some elements, any subset of elements.

It's the total # of subsets.

Size of this list: $\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}=16$

n elements			
<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>
y	n	y	n

(gives subset "AC")

The subsets are:

A,	AB,	CD,	ABCD.
B,	AC,	ABD,	
C,	AD,	ABC,	
D,	BC,	ACD,	
{ } null set,	BD,	BCD,	

Q) How many subsets does an n-element set have?

Two ways to answer this question.

Method 1:
Consider various sizes of subsets

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots\dots\dots\dots+\binom{n}{n}$$

Method 2:
Magic trick:
For each elements, decide
Whether it is in the subset or not.

It gives 2^n.

Because the two equations count the same thing, the two quantities must be equal.

H.W. # 4 due on 10/29. See website for assignment.