

Discrete Mathematics – Day 12-Sept. 26, 2003

Extra Credit HW:

*Try smaller problems and look for a pattern. Be systematic in your listing and counting.

Q: How many X-O strings of length 10 are there having no two 'O's next to each other?
Use Day 10 notes to find the pattern:

A:

```

      X,      O
      /\      \
      X, O    X
      /\  \   /\
      X,O X   XO
      ^  \  /\  /\  \
      X,O X X O X O X
    
```

Etc. (Every time you see an X you get an X,O, every time you see O you just get X)

- 1: X,O
- 2: XX, XO, OX
- 3: XXX, XOX, OXX, XXO, OXO
- 4: XXXO, XXXX, XO XO, XOXX...

Length	# of Ways	
1	2	$n_1 = 2$
2	3	$n_2 = 3$
3	5	$n_3 = 5$
4	8	$n_4 = 8$
5	13	
6	21	
7	34	
8	55	
9	89	
10	144	

Binet's Formula for Fibonacci #'s: $F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$

In our case, the # of ways for n X's and O's is given by the $(n+1)$ 'st Fibonacci number.

Fibonacci Numbers:

Def: $F_0 = 1, F_1 = 1$

$F_n = F_{n-1} + F_{n-2} \Rightarrow$ sum of 2 previous numbers.

For $n = 2$

F_0	F_1	F_2	F_3	F_4	F_5	F_6	$F_7...$
1	1	2	3	5	8	13	21...



Real Life Example:

Ex: Pine Cone \Rightarrow If you count # of spirals around pinecone, how many do you think there are? - # of spirals is always a Fibonacci #. (There are occasional "mutant" pinecones, but they are the exception.)

Group Work

Q: In how many ways can the 2 x 12 check board be dominized?

A:

2 x 1		1
2 x 2		2
2 x 3		3
2 x 4		5
.		8
.		13
		21
2 x 12		233

Length 5

All X's	XXXXX	
4X's	OXXXX	XXXOX
	XOXXX	XXXXO
	XXOXX	
3X's	OXOXX	XOXOX
	OXXOX	XOXXO
	OXXXO	XXOXO
2X's	OXOXO	
1X	-----	
0X	-----	

Gives 13 ways altogether