Notes for Discreet Math

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1 Exam 1

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2 Fibonnaci Numbers

$$F_n = F_{n-1} + F_{n-2}, F_0 = F_1 = 1$$

Example: 1,1,2,3,5,8,13,21,34,...

X O: 2,3,5,8,13,21,... combinations

Dominoes: 1,2,3,5,8,13...

How do we know this will continue forever?

How do we know that each number is the sum of the two before it?

3 Proof for X/O

Let X_n be the number of X/O sequences having no two Os adjacent. *By hand, we can find the number of X/O sequences of length n. We find that $X_1 = 2$, $X_2 = 3$, $X_3 = 5$. Now, for $n >= 4$, lets show that $X_n = X_{n-1} + X_{n-2}$* X_n is the number of strings with n letters having no two Os next to each other, so a string of length n can start with either X or O. Suppose a strings start with X and b strings starts with O. $n = a + b$. *Number of strings with length n that with O = ?*

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Example: O----------
         OX---------
           [       ]
            n - 2 characters
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How many ways to fill the remaining n-2 characters? X_{n-2} *The number of strings of length n is* X_{n-1} *How many strings of length n that start with X?*

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X___________--
 [n - 1 chars]
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There are X_{n-1} ways to finish building that string. The total number of ways to build the string is the number of ways to finish with X and the number of ways to to finish with O. $X_n = X_{n-1} + X_{n-2}$

4 Dominize

D_n is the number of ways to dominize a $2 * n$ board. $D_n = D_{n-1} + D_{n-2}$.

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+-+-+-+-+-+-+-+-+-+-+-+-+
| | | | | | | | | | | | |
+-+-+-+-+-+-+-+-+-+-+-+-+
| | | | | | | | | | | | |
+-+-+-+-+-+-+-+-+-+-+-+-+
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The top left square can be covered by either a vertical or horizontal dominoe. If it is a vertical dominoe, the rest can be covered in D_{n-1} ways. If it is horizontal, the rest can be coverted with D_{n-2} ways. The total number of ways is $D_n = D_{n-1} + D_{n-2}$

5 Homework 3

A) Write up a formal proof, that for dominoes, $D_n = D_{n-1} + D_{n-2}$

B) How many ways are there to "dominize" a $6 * 20$ checkerboard with $2 * 3$ dominoes.

1. Due: On exam day, worth 5 points per question.

6 Product Rule

A. How many scrabled positions are possible starting from the home position with the blank in the lower right by sliding the pieces only?

A. Naive answer: 15! Correct: $\frac{15!}{2}$

B. Start with a solved Rubik's cube, then make a number of random moves to gerate a scrambled pattern.

B. As an integer: 43,252,003,274,489,856,000

C. Same as B, on a 2 x 2 x 2 cube.

C.