A MODULAR REWRITING APPROACH TO LANGUAGE
DESIGN, EVOLUTION AND ANALYSIS

BY

MARK A. HILLS
B.S., Western Illinois University, 1995

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Doctoral Committee:
Associate Professor Grigore Roșu, Chair and Director of Research
Professor Carl Gunter
Associate Professor Samuel N Kamin
Professor José Meseguer
Professor Peter D Mosses, Swansea University
Abstract

Software is becoming a pervasive presence in our lives, powering computing systems in the home, in businesses, and in safety-critical settings. In response, languages are being defined with support for new domains and complex computational abstractions. The need for formal techniques to help better understand the languages we use, correctly design new language abstractions, and reason about the behavior and correctness of programs is now more urgent than ever.

In this dissertation we focus on research in programming language semantics and program analysis, aimed at building and reasoning about programming languages and applications. In language semantics, we first show how to use formal techniques during language design, presenting definitional techniques for object-oriented languages with concurrency features, including the Beta language and a paradigmatic language called KOOL. Since reuse is important, we then present a module system for K, a formalism for language definition that takes advantage of the strengths of rewriting logic and term rewriting techniques. Although currently specific to K, parts of this module system are also aimed at other formalisms, with the goal of providing a reuse mechanism for different forms of modular semantics in the future. Finally, since performance is also important, we show techniques for improving the executable and analysis performance of rewriting logic semantics definitions, specifically focused on decisions around the representation of program values and configurations used in semantics definitions.

The work on performance, with a discussion of analysis performance, provides a good bridge to the second major topic, program analysis. We present a new technique aimed at annotation-driven static analysis called policy frameworks. A policy framework consists of analysis domains, an analysis generic front-end, an analysis-generic abstract language semantics, and an abstract analysis semantics that defines the semantics of the domain and the annotation language. After illustrating the technique using SILF, a simple imperative language, we then describe a policy framework for C. To provide a real example of using this framework, we have defined a units of measurement policy for C. This policy allows both type and code annotations to be added to standard C programs, which are then used to generate modular analysis tasks checked using the CPF semantics in Maude.
To Sally.
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Chapter 1

Introduction

Software is a pervasive presence in our lives. Computers are now a common feature in homes and businesses, while smaller computers, known as embedded systems, are now found in everything from household appliances to phones to automobiles and airplanes. With the need for software growing, new languages are being defined to meet new development challenges. These languages often include support for complex abstractions, and are targeted at challenging domains such as families of configurable software products and ultra-large scale systems made up of distributed software components. Even in home computing systems, Internet connections and multi-core processors are now common, making once more academic concerns, such as concurrency and distributed computing, into concerns for regular application developers and game designers.

The pervasive nature of computation unfortunately has its downsides: computer failures can now be quite costly. This is often just measured in monetary costs: the loss of the NASA Mars Climate Orbiter due to a simple (yet hard to catch) programming error \[\text{2}\] resulted in a loss of 327.6 million dollars between the cost of the orbiter and associated lander \[\text{3}\]. Some software errors have led to consequences more serious than just loss of money: errors in the software used to control the Therac-25 radiation therapy machine \[\text{106}\] led to six known accidents, causing serious injuries and, in three cases, death.

The research described in this thesis is aimed around a core motivating concept: with the increased complexity of programming languages and software systems, along with the pervasive presence of software in everyday devices and safety-critical systems, the need for formal techniques to help better understand the languages we use, correctly design new language abstractions, and reason about the behavior and correctness of programs is more urgent than ever. One branch of this research is focused on programming language semantics, specifically on improving the ability to formally design new programming languages and extend existing languages with new features. The second branch of the research described herein is focused on program analysis and program verification, specifically on using semantics-driven techniques to find errors in programs and/or show that they are correct in regards to a given specification.

Unfortunately, formal techniques for defining the semantics of a programming language have not been very successful outside the research community, and
often are not used even inside the research community[136,140]. An overall goal across both branches of this research is to contribute to changing this, by providing better ways to define language features, reuse defined features across languages, and analyze programs based on a given language semantics.

1.1 Contributions

The research described in this thesis makes the following key contributions:

1. This thesis gives the first rewriting logic semantics and K definitions of a number of complex features found in real programming languages, including Smalltalk-like primitive operations (used to perform operations such as arithmetic or value comparisons in pure object-oriented languages without scalars), Beta-style inner calls, auto-boxing of scalars into objects, coroutines, and garbage collection. This research is described in detail in Chapters 3, 4, and 6.

2. Current K tool support requires language feature definitions to be manually assembled into a single module before use. This thesis introduces a module system for K, providing a mechanism to build modules containing reusable language features, and including novel features designed for making definitions more concise and for working with different kinds of semantics (standard dynamic and static semantics, semantics aimed at program analysis, etc.). The module system also includes additional functionality for sharing modules between different tools and over the Internet, with a shared module repository. This research is described further in Chapter 5.

3. To improve the flexibility of program analysis frameworks, we introduce policy frameworks, the first mechanism to define generic, modular analysis frameworks with a focus on reuse both within a single language and (without a translation into a shared intermediate language) across multiple languages. As a proof of concept, we also introduce the C Policy Framework, including a policy for checking the proper usage of units of measurement, the UNITS policy. This policy is competitive with existing state of the art tools for checking for unit safety, with good performance and an annotation language more expressive than those provided by any other annotation-based unit checker of which we are aware. It also achieves a large amount of reuse, with a significant portion of the definition shared with other policies in CPF and with the units domain shared with policy frameworks for other languages. Policy frameworks are described in more detail in Chapter 7, while the C Policy Framework and the UNITS policy are described further in Chapter 8.

The contributions listed above have primarily been driven by this author, although all are to some extent collaborative. The basis for this work has been
the computation-based style of rewriting logic semantics and K (described more in Chapter 2), developed by Grigore Roșu. The definition of Beta has been a joint effort with Barış Aktemur, while some of the core research directions for policy frameworks and the UNITS policy were developed in collaboration with Grigore Roșu and Feng Chen. Work on the module system has involved close cooperation with Grigore Roșu; Traian Florin Şeşerău who has been working on the K Tools developed in Maude; and Chucky Ellison, who developed an initial (non-modular) toolset for working with K and is an active user of the new module system.

**Broader Impact:** The broader impact of these contributions is in the following areas:

1. Research into defining complex language features using K benefits both language designers and other researchers in language semantics, providing definitions which can be used directly during language prototyping and for guidance when defining new, but similar, features.

2. Research into the K module system makes some initial steps towards having a shared repository of reusable language features, a goal not just for K but for other styles of semantics as well (such as the work on Component-Based Semantics, discussed in Chapter 9). It also makes the work on K more accessible to others, providing, as the repository is loaded, a number of pre-built and pre-tested features which are ready to be used when building language definitions.

3. Research on policy frameworks provides a general method for adding reusable analysis frameworks to languages, hopefully moving the creation of analysis tools away from highly focused tools that are not reusable or extensible and allowing semantics-based analysis frameworks to be added more easily to a language. The work on policy frameworks is also currently acting as a springboard for work on proving properties of programs in a modular fashion.

As mentioned above, one goal of this research is to increase the use of formal techniques in language design. It is hoped that, by providing tools and techniques which can be used to prototype even complex languages; ways to leverage these definitions for program analysis; and tool support for working with existing feature definitions, the research outlined in this thesis will lead to an increased use of formal techniques (in general) and K (specifically) for language design. It is also hoped that the work on the module repository, including a standard exchange format for language feature definition modules and tool support for both interacting with the repository and combining modules, will also be useful for modular semantics formalisms other than K, such as MSOS, Action Semantics, or Monads.
1.2 An Overall Guide to the Thesis

Chapter 2 provides a brief introduction to some background material that is helpful to understanding the remainder of the thesis: term rewriting systems, equational and rewriting logic, rewriting logic semantics, and K.

Chapters 3, 4, and 5 are focused on language semantics: Chapter 3 discusses language prototyping in the context of the KOOL language, while Chapter 4 widens this discussion to include features of the Beta programming language. Chapter 5 then provides details on the K module system, including an online module repository designed to hold not only K modules, but modules defined in other formalisms as well.

Chapters 6, 7, and 8 are focused on program analysis. Chapter 6 acts as a bridge of sorts, tying some of the work on language design in with both execution and analysis performance. Chapter 7 then introduces the concept of policy frameworks, using the SILF language as an illustrative example. The C Policy Framework, built around the same principles as the policy framework for SILF, but with a much more complex language and a keener focus on performance, is then described in Chapter 8.

The next chapter, Chapter 9, discusses related work, especially focusing on work in tool-supported semantics, definitional modularity, and program analysis. An in-depth comparison of K with other formalisms is not presented, but is itself the topic of a fair portion of the current K report. The thesis then ends with conclusions and a discussion of planned and possible future research based on the topics presented in this thesis, found in Chapter 10. Cited references, along with an appendix including a current Vita for the author, are included at the end of the thesis.

1.3 Relationship to Previous Work

The work on rewriting logic semantics and K has similar goals to other work on language semantics – providing a means to define and reason about programming languages and their programs. One goal of this research has been to overcome some of the shortcomings we found in other formalisms, while taking advantage of the algebraic setting provided by rewriting logic. A specific goal has been to create modular definitions made up of reusable pieces, a goal shared by a number of other formalisms, such as MSOS, Action Semantics, Monads, and (with Montages) Abstract State Machines. This has driven features of K, such as context transformers (described in Chapter 2), and features of the module system (described in Chapter 5). A comparison of this work on language prototyping and modularity with similar work in other formalisms is provided in Chapter 9.

The work on policy frameworks grew out of this work as a method to leverage the modularity of definitions towards creating reusable analysis frameworks. Some of the concepts were based on earlier work on using rewriting logic se-
mantics for program analysis, and the ideas are similar to those from other analysis frameworks, such as JML and Frama-C (but with a focus on multiple programming languages). Checking the safety of programs that use units of measurement has been a major driver of the work, with a goal of providing a unit checker better than both our own prior work in the area and work on competing solutions, such as solutions based on program libraries or on the use of an analysis tool such as Osprey. Chapter 9 provides a comparison between policy frameworks and other analysis frameworks, with a special focus just on units of measurement.

1.4 Related Publications

This section provides a quick overview of this author’s publications, explaining their relationship to the contents of this thesis.

**K:** The first appearance of K was in *A Rewrite Framework for Language Definitions and for Generation of Efficient Interpreters* [78]. Although K is used in a number of other papers, its next feature appearance was in *Towards a Module System for K* [85]. K is used throughout the thesis, starting in Chapter 2 while the material on the module system is presented, in expanded form, in Chapter 5. Chapter 5 also presents information on a module repository and a shared exchange format for language feature modules, both of which are related to the module system but are new to this thesis.

**KOOL and SILF:** KOOL, presented in Chapter 3, was first discussed in *An Application of Rewriting Logic to Language Prototyping and Analysis* [83]. Around the same time, information on improving the performance of KOOL for verification, found in Chapter 6, was published in *On Formal Analysis of OO Languages using Rewriting Logic: Designing for Performance* [84]. A number of technical reports, including *A Rewrite Logic Approach to Semantic Definition, Design and Analysis of Object-Oriented Languages* [30], *KOOL: A K-based Object-Oriented Language* [82], and *A Rewriting Based Approach to OO Language Prototyping and Design* [84], further expanded this work. The material on KOOL in this thesis is based directly on all of these, with the semantics presented here reformulated to use the latest version of the K notation.

Like K, SILF was first introduced in *A Rewrite Framework for Language Definitions and for Generation of Efficient Interpreters* [78]. The material on SILF is mainly present as background for Chapter 6. KOOL and SILF then made a joint appearance in *Memory Representations in Rewriting Logic Semantics Definitions* [73], which discussed the relationship between different memory models – including a garbage collector for KOOL – and performance. This material is also discussed here in Chapter 6.
**Beta:** Beta, discussed in Chapter 4, was first defined using rewriting logic semantics in *An Executable Semantic Definition of the Beta Language using Rewriting Logic* [75]. The presentation here is mainly based on this technical report, but with rules given using K notation. Chapter 4 also mentions a current reformulation of the semantics, which has not been presented elsewhere.

**Policy Frameworks:** Policy frameworks first appeared in a technical report, *Pluggable Policies for C* [77]. This work initially focused on a framework for C, presented here in Chapter 8. *A Rewriting Logic Approach to Static Checking of Units of Measurement in C* [76] focused specifically on units of measurement analysis using CPF: units are used here as an example in Chapters 7 and 8 with a focus on C in Chapter 8. An earlier approach to units analysis was presented in *Automatic and Precise Dimensional Analysis* [36], but does not make a direct appearance here. The SILF Policy Framework, based on work on both SILF and the C Policy Framework, is new to this thesis.
Chapter 2

Background

This chapter provides an introduction to equational logic, term rewriting, rewriting logic, rewriting logic semantics, and K. Equational logic, introduced in Section 2.1, provides a method for reasoning about equalities between terms, which in our case are used to represent programs and semantic configurations. Term rewriting, introduced in Section 2.2, represents computations by the progressive transformation of terms according to term rewriting rules, providing a method of executing equational logic definitions. Section 2.3 introduces rewriting logic \cite{117,114}, an extension of equational logic with support for reasoning about nondeterministic and concurrent computation. Rewriting logic can be used to define the semantics of sequential and concurrent programming languages, leading to a form of semantics called rewriting logic semantics \cite{120,121}, introduced in Section 2.4. Finally, Section 2.5 describes K \cite{155}, a method, based on the work on rewriting logic semantics, for formally defining programming languages. This introduction focuses on that background needed specifically to understand the research presented in this thesis; additional information on each of these topics can be found in the references cited throughout this chapter.

2.1 Equational Logic

Equational logic is a logic for reasoning about equational theories, also called algebraic specifications \cite{192}. An equational theory is made up of two parts: $\Sigma$, the signature, which defines the syntax provided to form terms; and $E$, a set of equations between $\Sigma$-terms.

2.1.1 Signatures

$\Sigma$ contains a set $S$ of sorts, which indicate the types of terms. Sorts can represent standard mathematical entities, such as Nat, Real, or Set, but can also be used to define entities used in programming language semantics, such as Expression, Statement, Value, Program, Environment, or Continuation. Using Maude \cite{33} syntax, these are defined using the sorts keyword:

\begin{verbatim}
sorts Nat Expression Value .
\end{verbatim}
If \( S \) contains just one sort \( \Sigma \) is referred to as unsorted or single-sorted. Signatures used to represent languages usually contain multiple sorts, in which case \( \Sigma \) is a many-sorted signature. It is also possible to include an order relation, \(<\), between sorts, where, given two sorts \( s \) and \( s' \), \( s < s' \) indicates that the terms of sort \( s \) are also terms in \( s' \) – for instance, \texttt{Nat} < \texttt{Int}. This order relation is a partial order \[32\] – it is transitive, but not symmetric. Again using Maude notation:

\[
\text{sorts Nat Int Rat}.
\]
\[
\text{subsort Nat < Int < Rat}.
\]

Along with sorts, \( \Sigma \) also contains operations, which provide the syntax used to form terms. In Maude, operations are defined using the \texttt{op} keyword:

\[
\text{op zero : } 	ext{-> Nat}.
\]
\[
\text{op succ : Nat } 	ext{-> Nat}.
\]
\[
\text{op plus : Nat Nat } 	ext{-> Nat}.
\]

Operations are given a name, like \texttt{zero} or \texttt{plus}, and, following \( : \), a signature indicating the number of arguments, the sort of each argument, and the result sort, given after the arrow. Operations require 0 or more arguments, with 0-argument operators used to represent constants. Above, \texttt{zero} takes no arguments (making it a constant), \texttt{succ} takes one, and \texttt{plus} takes two. The operators shown here are defined as prefix operators, meaning the operator will come before its arguments, given in parentheses. Sample terms over these operators include the following:

\[
\text{zero}
\]
\[
\text{succ(succ(zero))}
\]
\[
\text{plus(succ(zero),succ(succ(zero)))}
\]

The first term uses the constant \texttt{zero}; the second represents two as the successor of the successor of \texttt{zero}; and the third represents the addition of one and two.

Operations can also be defined in a mixfix form, which allows the operators to be used more like standard programming language syntax:

\[
\text{op 0 : } 	ext{-> Nat}.
\]
\[
\text{op s_ : Nat } 	ext{-> Nat}.
\]
\[
\text{op _+_ : Nat Nat } 	ext{-> Nat}.
\]

Argument positions in mixfix operators are indicated by the position of \( _ \), with \( s \) including a single argument and \( + \) including two, one before the \( + \) character and one after. The same terms as given above using prefix operators would be represented as follows using mixfix operators:

\[1\] It is possible to assign precendences to the defined operators; we ignore that here, using parentheses to group parts of terms in cases where the meaning would otherwise be unclear to the reader.

8
Using the above as an aid to intuition, we can now give the following mathematical definition of a signature $\Sigma$:

**Definition 1** $\Sigma = \{S, \{\Sigma_{w,s}\}((w,s) \in S^* \times S^*), <\}$, where $S$ is a set of sorts, $\{\Sigma_{w,s}\}((w,s) \in S^* \times S^*)$ is an $S^* \times S^*$-indexed family of sets of operation symbols, and $<$ is a transitive, irreflexive, and antisymmetric order relation on $S$.

### 2.1.2 Algebras

The signature $\Sigma$ provides the syntax for the equational theory, but the syntax does not provide a semantics – it assigns no meaning to the terms. The mathematical meaning, or model, of a signature is provided by $\Sigma$-algebras.

**Definition 2** Given many-sorted signature $\Sigma$, $\Sigma$-algebra $A$ is defined by: an $S$-indexed family of sets $A = \{A_s\}_{s \in S}$, called the carrier of the algebra; an element $a^s_A \in A_s$ for each constant $a : \rightarrow s$ in $\Sigma$; and a function $f^{w,s}_A : A_{s_1} \times \cdots \times A_{s_n} \rightarrow A_s$ for each operation $f : w \rightarrow s$ in $\Sigma$ (where $w = s_1 \ldots s_n$ and $n > 0$).

We do not show the definition for algebras over order-sorted signatures here. They are similar to that shown for many-sorted signatures, with some additional requirements which ensure that constants present in multiple sorts related by $<$ represent the same value in all carriers (e.g., $0$ has the same meaning as a natural number, integer, and rational number) and that operations which are redefined on sorts related by $<$ agree on the result when given the same argument values (e.g., addition over naturals, over integers, and over rationals should all agree on the result when given the same natural number arguments).

**Term Algebras:** A particularly important algebra, referenced later in this thesis, is the term algebra, $T_\Sigma$. This algebra contains all well-formed terms produced over the syntax of $\Sigma$. In a programming language context, $T_\Sigma$ contains all syntactically valid programs in the language whose syntax is defined by $\Sigma$. Using the operations defined earlier for natural numbers, $T_\Sigma$ would include $0$, $s \ 0$, $s\ s\ 0$, $\ldots$, $0 + 0$, $0 + (s \ 0)$, $(s \ 0) + 0$, etc.

### 2.1.3 Equations

Equations are used to indicate when two terms are equal. An (inadvisable) example would be to say that the natural numbers $1$ and $0$ are equal:

$$\text{eq } s\ 0 = 0 .$$
Equations can include variables, representing arbitrary terms over the sort of the variable. When present, variables are considered to be universally quantified over the equation. The following equation indicates that addition is commutative:

\[
\text{eq } X + Y = Y + X.
\]

This assumes that both \(X\) and \(Y\) are declared to have sort \(\text{Nat}\), and would be written mathematically as

\[
(\forall X Y) \ X + Y = Y + X
\]  \hspace{1cm} (2.1)

Note that, with the introduction of variables, given two terms \(t\) and \(t'\) it is not possible to just compare \(t\) and \(t'\) syntactically to determine if, using this equation, \(t\) and \(t'\) are equal. For instance, one may want to determine if the following equality holds:

\[
(s \ s \ s \ 0) + (s \ 0) =? (s \ 0) + (s \ s \ s \ 0)
\]

To do so, one must first find a substitution, \(\theta\), mapping the variables in the equation to subterms of \(t\) (here \((s \ s \ s \ 0) + (s \ 0)\)) and \(t'\) (here \((s \ 0) + (s \ s \ s \ 0)\)). Using \(\tilde{\theta}\), the homomorphic extension of \(\theta\) to a function from terms to terms \(^3\) it is then possible to see if two terms \(t\) and \(t'\) are equal under an equation \(u = u'\) by applying \(\tilde{\theta}\) to both \(u\) and \(u'\), \(\tilde{\theta}(u) = \tilde{\theta}(u')\), and verifying that either \(\tilde{\theta}(u) = t\) and \(\tilde{\theta}(u') = t'\) or \(\tilde{\theta}(u) = t'\) and \(\tilde{\theta}(u') = t\). In this example, \(\tilde{\theta}(X) = s \ s \ s \ 0\), \(\tilde{\theta}(Y) = s \ 0\), \(\tilde{\theta}((s \ s \ s \ 0) + (s \ 0)) = (s \ s \ s \ 0) + (s \ 0)\), and \(\tilde{\theta}((s \ s \ s \ 0) + (s \ 0)) = (s \ 0) + (s \ s \ s \ 0)\), so the two terms \(t\) and \(t'\) are shown equal by the commutativity of addition.

Equations can also have conditions, which indicate that the equation only holds when the conditions are fulfilled. Conditions are specified with the keyword \(\text{if}\) in Maude syntax and the symbol \(⇐\) outside of Maude. For instance:

\[
\text{ceq } X + Y = Y \text{ if } X == 0.
\]

specifies that 0 is the (left) identity for the addition of natural numbers, and could also be written \((\forall X Y) \ X + Y = Y ⇐ X == 0\).

Using equations it is possible to make one step deductions, but it would not be possible to show that two terms are equal if establishing equality requires the use of multiple equations. This is the purpose of the equational logic deduction system, which consists of the following rules over unconditional equations. Here \(t\), with or without primes and subscripts, is used to represent arbitrary terms formed over \(\Sigma\); while \(X\) is a set of variables, instead of a single designated variable (as it was above):

\[
(\forall X)t = t \hspace{1cm} (\text{Reflexivity})
\]

\(^2\)This style of writing equations, with explicit quantifiers, was first used in [58], and has since been used elsewhere, including in the context of defining languages [61].

\(^3\)The simplest way to view this is that \(\tilde{\theta}\) recurses over the structure of a term, applying \(\theta\) to any variables it finds.
\[
\frac{(\forall X)t = t'}{(\forall X)t' = t^\prime} \quad \text{(Symmetry)}
\]

\[
\frac{(\forall X)t = t' \quad (\forall X)t' = t''}{(\forall X)t = t''} \quad \text{(Transitivity)}
\]

\[
\frac{(\forall X)t_1 = t'_1 \quad \ldots \quad (\forall X)t_n = t'_n}{(\forall X)f(t_1, \ldots, t_n) = f(t'_1, \ldots, t'_n)} \quad \text{(Congruence)}
\]

An additional rule is used specifically for conditional equations, formalizing the notion mentioned above that the equation applies only when the condition is true. Given equation \((\forall X) t = t' \iff u_1 = v_1 \land \ldots \land u_n = v_n\), where \(u_1 = v_1\) through \(u_n = v_n\) are the conditions (with \(\land\) logical and) and \(X\) and \(Y\) are sets of variables:

\[
\frac{(\forall Y) \theta(u_1) = \theta(v_1) \quad \ldots \quad (\forall Y) \theta(u_n) = \theta(v_n)}{(\forall Y) \theta(t) = \theta(t')} \quad \text{(Modus Ponens)}
\]

A Note on Algebras: As discussed above, the model of a signature \(\Sigma\) is an algebra, providing sets of values for each sort, a value in these sets for each constant, and a function for each operation. The models for an equational theory \((\Sigma, E)\) are also algebras, with the additional restriction that only those algebras in which all equations in \(E\) hold are models of \((\Sigma, E)\).

2.1.4 Equational Theories in Maude

Maude captures the concept of an equational theory using a functional module, declared using the \texttt{fmod} keyword and containing both the signature of the theory and any equations. Figure 2.1.4 shows an example of a functional module that defines natural numbers. \texttt{Nat} is declared as a sort. The operators then define both the constructors (0 and s) for natural numbers and an extra operation

\[
\text{fmod NAT is}
\]

\[
\text{sorts Nat .}
\]

\[
\text{op 0 : -> Nat .}
\]

\[
\text{op s_ : Nat -> Nat .}
\]

\[
\text{op _+_ : Nat Nat -> Nat .}
\]

\[
\text{vars X Y : Nat .}
\]

\[
\text{eq s(X) + Y = s(X + Y) .}
\]

\[
\text{eq 0 + X = X .}
\]

\[
\text{eqdfm}
\]

Figure 2.1: A Sample Functional Module, in Maude

11
for addition, +. vars defines two variables, $X$ and $Y$, both representing natural numbers. Finally, two equations are defined using eq: the first gradually moves all successors “out”, while the second specifies that 0 is the left identity (but without using the more cumbersome conditional equation shown above).

## 2.2 Term Rewriting

Equational logic provides support for defining terms and reasoning about term equalities. However, it does not provide a method for computing with terms. This is provided by term rewriting [12].

In term rewriting, a number of rewrite rules are defined. These rules, of the form $l \rightarrow r$, are used to progressively change the term being rewritten. Like equations in equational logic, rules can contain variables. To determine which rule to apply next, the rewrite engine uses a process called matching. The rule matches if a substitution can be found such that, after substituting terms for the variables in the left hand side of the rule, the left hand side matches the current term or one of its subterms. Mathematically, given a subterm $t'$ of term $t$ (where $t$ could equal $t'$), rule $l \rightarrow r$ matches if a substitution $\theta$, from variables to terms, can be found such that $\theta(l) = t'$. If a match is found, $t'$ is rewritten to $\theta(r)$. When no more matches can be found, the final term is the result of the computation. Since term rewriting systems are Turing complete, it is possible that the computation will not terminate (i.e., that it will always be possible to apply another rule).

The equations defined in equational logic can be used as term rewriting rules by orienting them, changing an equation defined as $l = r$ into a rewrite rule $l \rightarrow r$. This can be problematic in some cases. For instance, if equations are used to define that an operator is commutative, the rewriting process could diverge, continually swapping the positions of terms without making any progress. Because of this, Maude (as well as other systems) allows operators to be defined with attributes that indicate an operator is associative, commutative, and/or has an identity (such as the empty set in a set formation operation), but only during matching. This allows the rewrite engine to (for instance) treat an operator as commutative when deciding which rule to apply, but it prevents the rewrite engine from applying commutativity as a rule directly. Without a commutative attribute, an operator would be defined as commutative by including an additional equation:

\[
eq X + Y = Y + X.
\]

This would yield a rewrite rule which would cause an endless series of “flips” around the plus:

\[
X + Y \rightarrow Y + X
\]
Using the commutative attribute in Maude, this would instead be defined as:

\[
\text{op } _+_: \text{Nat Nat } \rightarrow \text{Nat \ [comm]}.
\]

Associativity, commutativity, and identity attributes are used heavily in the definitions described in this thesis, because they provide a natural way to define lists and sets (including multisets), both of which are regularly used in formal language definitions. A list is defined using an associative list formation operator and an identity, representing the empty list:

\[
\begin{align*}
\text{op empty : } & \rightarrow \text{List}. \\
\text{op } _,_ &: \text{List List } \rightarrow \text{List \ [assoc id: empty]}.
\end{align*}
\]

This is also one of the main uses of subsorting in the definitions described in this thesis, allowing an item of a given sort to be treated as a trivial list (or set) of one element:

\[
\text{subsort Nat } < \text{NatList}.
\]

Using an identity provides a way to eliminate corner cases. Without an identity, an operation to return the head of a list would be defined as follows:

\[
\begin{align*}
\text{op hd : NatList } & \rightarrow \text{Nat} . \\
\text{eq hd}(X) &= X . \\
\text{eq hd}(X,Xs) &= X .
\end{align*}
\]

The first equation is needed for the trivial case, where the list is made up of just a single item. The second equation handles the more standard case, where the list includes a head and a tail. Using the identity, the matching process can always “add” an implicit (empty) tail to the list for matching purposes, allowing the operation to be defined with just one equation:

\[
\begin{align*}
\text{op hd : NatList } & \rightarrow \text{Nat} . \\
\text{eq hd}(X,Xs) &= X .
\end{align*}
\]

A set or multiset is instead defined using an associative, commutative set formation operator. A standard definition of a multiset of natural numbers would be:

\[
\begin{align*}
\text{sorts NatSet} . \\
\text{subsort Nat } & < \text{NatSet} . \\
\text{op nil : } & \rightarrow \text{NatSet} . \\
\text{op } _,_ &: \text{NatSet NatSet } \rightarrow \text{NatSet \ [assoc comm id: nil]}.
\end{align*}
\]

This definition treats juxtaposition as set formation, and allows the elements in the set to be rearranged at will for matching purposes. Like in the list example given above, the use of an identity also eliminates corner cases. For instance, a membership test for a set, without an identity, would be written as follows:
op _in_ : Nat NatSet -> Bool.
eq N \text{ in } N = \text{true}.
eq N \text{ in } N \text{ NS} = \text{true}.
eq N \text{ in } M = \text{false} \text{ [owise]}.neq N \text{ in } M \text{ NS} = \text{false} \text{ [owise]}.

Note the use of [owise] here, which says that the given equation applies when the others do not. This ensures the last two equations only hold when $N$ and $M$ are not the same number (if $N$ and $M$ are the same, one of the first two equations would hold instead). Using an identity, the special case, where the set consists of only one element, can be removed:

\begin{align*}
op \_\text{in}_\_ : \text{Nat NatSet} & \to \text{Bool} . \\
eq N \text{ in } N \text{ NS} & = \text{true} . \\
eq N \text{ in } \text{NS} & = \text{false} \text{ [owise]} .
\end{align*}

A large number of term rewrite engines are currently in use, including ASF+SDF [179 180], Elan [19], Maude [33], OBJ [59], and Stratego [185 22]. Rewriting is also a fundamental part of existing languages, including Tom [13 131 104], which integrates rewriting with Java.

### 2.3 Rewriting Logic

Using equational logic, it is possible to model many deterministic systems, creating operations to represent the state of the system and equations to represent how the system can evolve. Based on the rules of deduction for equational logic, this means that all system states that are provably equal can be considered to be the same, or, more accurately, all states that are provably equal are members of the same equivalence class of terms modulo the equations in $E$, meaning any one of the terms in the class can be chosen as a representative for all the other terms.

Switching to the term rewriting perspective, given a starting term $t$ (say $(s s s 0) + (s 0)$), the final term $t'$ (here $s s s s 0$) conceptually represents the same entity. This is the same perspective taken in the lambda calculus, where terms can be grouped into equivalence classes based on $\alpha$ and $\beta$ equivalence, with a term then in the same equivalence class as its fully reduced form.

One limitation of this is that it is not possible to represent transitions between terms that do not lead to equivalent terms. This is the case in systems that have nondeterminism or actual concurrency, such as Petri nets and programming languages with threads. For instance, in a Petri net [150 119], transitions change the distribution of tokens in the net, potentially preventing other transitions from firing or allowing new transitions to be active. In a programming language, updates to shared memory locations in different threads can compete, potentially changing the final result of a computation based on the order in which the threads execute.
Rewriting logic \[117\] is an extension of equational logic with support for reasoning about nondeterminism and (more broadly) concurrency:

**Definition 3** A rewrite theory \( \mathcal{R} \) is a triple \( \mathcal{R} = (\Sigma, E, R) \), with \( (\Sigma, E) \) an equational theory and \( R \) a set of labeled rewrite rules \( l : t \rightarrow t' \Leftarrow c \) where \( l \) is a label, \( t \) and \( t' \) are terms formed over \( \Sigma \), and \( c \) is a condition.

Like equations, rules can include variables and can be conditional. Unlike equations, rules cannot be read in both directions, which gives them the power to evolve one term into another which need not be equationally equal to the first. This can be seen as using rules to move between classes of terms modulo \( E \). Rules in rewriting logic map to rewrite rules in term rewriting systems directly, given that they are already oriented.

### 2.3.1 Rewrite Theories in Maude

Maude captures the concept of a rewrite theory using a system module, declared using the `mod` keyword. System modules are extensions of functional modules that also providing support for rules, declared using the keywords `rl` (for unconditional rules) and `crl` (for conditional rules). An unconditional rule is declared as:

\[
\text{rl [LABEL]} : l \Rightarrow r .
\]

where `LABEL` (which is optional) provides a name for the rule, and \( l \) and \( r \) are the left and right sides of the rule. Note the use of `\Rightarrow` instead of `\Rightarrow`; this is because rules are not equalities, and can only be used for reasoning from left to right. Conditional rules are declared similarly, but have a condition like that given with a conditional equation:

\[
\text{crl [LABEL]} : l \Rightarrow r \text{ if } c .
\]

---

Technically, \( t \) and \( t' \) are both of the same kind, meaning the sorts assigned to \( t \) and \( t' \) are related, potentially indirectly, by the ordering relation \( < \) discussed above.

```maude
mod CANDY-AUTOMATON is
  sorts State.
  ops \$ ready broken nestle m&m q : -> State.
  rl [in] : \$ => ready .
  rl [cancel] : ready => \$ .
  rl [chng] : nestle => q .
  rl [chng] : m&m => q .
endm
```

---

Figure 2.2: A Nondeterministic System Module, in Maude
**mod PETRI-MACHINE is**

**sorts Marking .**

**ops null $ c a q : -> Marking .**

**op _ _ : Marking Marking -> Marking [assoc comm id: null] .**

rl [buy-c] : $ => c .

rl [buy-a] : $ => a q .

rl [chng] : q q q q => $ .

**endm**

Figure 2.3: A Concurrent System Module, in Maude

Figure 2.2, borrowed from a course on program verification [116], shows an example of a nondeterministic vending machine formalized as an automaton. The rule in models the insertion of money into the machine. Once the vending machine is ready, several new transitions are enabled: the transaction can be canceled, two different snacks can be purchased, or the machine could be broken. Assuming an item is purchased, the vending machine issues a quarter (q) in change.

Figure 2.3, from the same course [116], models a Petri net. Various tokens, $, c, a, q, make up a “marking”, which gives the current state of the net. Markings are defined as sets, using the syntax shown in Section 2.2. Rules define how markings change: a $ can change to a c or an a q. Rules can also be applied concurrently: $ $ could change, in one step, to either c c, a q a q, c a q, or a q c (the latter two being equal).

### 2.3.2 Tool Support

Maude provides several tools, beyond a term rewriting engine, which have been used in the research presented in this thesis:

1. Maude provides the ability to perform a breadth-first search over the state space of a program, with states equivalence classes of terms modulo E and transitions between states determined by the application of rules. This capability can be used to find deadlocks (executions which “get stuck”) and to explore the evolution of a system as it runs. It can also be used to pose “what if?” questions, with the goal being to determine if a state of interest is reachable.

2. Maude also provides an LTL model checker. The model checker uses propositions defined over the state of the system (i.e., defined over the term, such as the marking for Petri nets shown above); these propositions are then included in LTL formulae. It is possible both to verify that certain properties hold in finite state systems and to find counterexamples where properties do not hold in either finite state or infinite state systems (although obviously in the latter the search for a counterexample may not terminate).
2.4 Rewriting Logic Semantics

Equational logic has long been seen as a viable formalism for defining the semantics of sequential programming languages [63, 64]. Rewriting logic extends this by providing a formalism for defining the semantics of nondeterministic and concurrent languages, leading to an area of research known as rewriting logic semantics [120, 121]. One specific style of rewriting logic semantics, which influenced the development of K (discussed in Section 2.5) and includes some of the work discussed in this thesis, is computation-based rewriting logic semantics.

2.4.1 Computation-Based Rewriting Logic Semantics

The computation-based style of rewriting logic semantics (hereafter RLS) defines the semantics of a programming language as a rewrite theory. The definition of the semantics is given in an operational style, with terms used to represent the current configuration – the current program and state – and rewriting logic rules and equations used to represent transitions between configurations.

Sorts and Operations in RLS

The signature Σ contains sorts and operators representing: the abstract syntax of the defined language; the configuration as a whole, as well as the various parts that make up the configuration (e.g., an algebraic definition of the concept of an environment); and the various auxiliary operations used as parts of the semantics, including the individual operations used in the definitions of various language features and the concept of a computed value. As a shorthand, this last category is referred to later as semantic entities. Note that the distinction between these three groups is arbitrary and made just to ease discussion: the sort and operator syntax shown above for Maude is used to define all three.

Abstract Syntax: Examples of abstract syntax from the KOOL language, discussed further in Chapter 3, are shown in Figure 2.4. First, a sorts declaration defines a new sort, Exp, representing expressions in the abstract syntax for KOOL. The subsort declaration specifies that terms of sort Name are also considered to be of sort Exp, similar to a BNF production like Exp ::= Name. Several operators then declare parts of the abstract syntax. The first, common in many
op empty : -> KState .

op cset : ClassSet -> KState .
op env : Env -> KState .
op k : Computation -> KState .
op t : KState -> KState .

Figure 2.5: RLS Configuration Definitions

languages, says that an expression can also be used as a statement, indicated by following it with a semicolon. The second defines the constant Nil, the null reference value for KOOL. The third defines a standard conditional, with an expression and two statements, one for each branch. The last defines a while loop, again with a condition and then the loop body. Defining the abstract syntax using mixfix notation provides a way to use notation which closely resembles the actual language constructs and which can easily be mentally converted into BNF – words in the operator are tokens, each underscore can be replaced by the sort it represents to fill in the nonterminals, and the sort after the arrow could be moved to the front before ::= . For instance, the operator declaration while_do_od becomes Stmt ::= while Exp do Stmt od.

Configuration Items: Configuration items are also defined as operators, and represent the same items used in other styles of semantics, such as environments, stores, and tables of information about functions, classes, methods, etc. Configuration operators generally all have the same target sort, but can have any argument sorts, holding individual pieces of information, arbitrary tuples, lists, sets, finite maps, and multisets. Several sample configuration items from KOOL, as well as the general declaration for configurations, are shown in Figure 2.5.

In Figure 2.5, all configuration items are given sort KState. KState itself is defined as a multiset: putting together two KStates forms a new KState, and during matching (i.e., when deciding which equations and rules to apply), individual KState items can be rearranged (comm) and grouped (assoc) as needed. This provides two advantages: configuration items do not need to be named in a specific order in equations and rules; and items that are not needed do not need to be included in the equation or rule, making the semantics more modular (a point discussed further both in Section 2.5 and Chapter 9). Multisets also allow the same configuration item to appear multiple times, which is useful for items (such as threads) that can be repeated and which could (in theory) have the same contents, but which should not be collapsed into the same item.

Four configuration items are then defined, cset, env, k, and t. cset holds a set of classes, used in KOOL to keep track of classes that have been defined; env holds the current environment, defined as a finite map, which maps the names of
variables to their locations in memory; and $t$ is used to represent the state local to an individual thread, defined as a multiset of other configuration items. This provides a natural way to model the fact that some information is local to (and present in) each thread, such as the current environment for a method running in the thread, while some information is global to the entire computation, like shared memory.

$k$ holds the current computation, and deserves special mention since it is a key part of the semantics. Computations in $k$ are lists; each item in the list is referred to as a computation item, each of which represents an individual task or piece of information in the computation. The head of the list can be seen as the “next” task, with the tail containing tasks that will be computed later. Instead of using “,” as the list separator, an arrow, written in text as $\rightarrow$ and mathematically as $\rightsquigarrow$, is used instead, hopefully providing some added intuition: do this (computation item $ci_1$), then that ($ci_2$), then that ($ci_3$), etc, until finished ($ci_n$ is finished):

$$ci_1 \rightsquigarrow ci_2 \rightsquigarrow ci_3 \rightsquigarrow ... \rightsquigarrow ci_n$$

The equations and rules used to define the semantics (discussed below) often break up computations into smaller pieces, which are then put at the head of the computation to indicate that they need to be computed first before the overall computation can continue. Computations in RLS are just first-order terms, making them easy to manipulate, for instance by saving the current computation to resume later (for coroutines) or by creating a computation to act as an exception handler.

The configuration item defined in Figure 2.5 are actually part of a much larger configuration for KOOL, shown in Figure 2.6 and discussed further in Chapter 3. Configurations in RLS are often hierarchical, with some parts of the configuration (such as the current computation, the environment, etc) nested inside other parts of the configuration (such as individual threads). As mentioned above, this provides a natural way to duplicate parts of the configuration where needed. It also provides a way to group related pieces of information and then refer to them as a unit, instead of having to refer to each piece of information individually.

**Semantic Entities:** Finally, semantic entities are defined similarly to abstract syntax and configuration items. Figure 2.7 shows several examples: iv, which “injects” integers into the sort Value, indicating that integers are valid values (i.e., results of computations) in KOOL; a definition of ObjEnv, or object environments, used to track mappings from names to locations at each allocated “level” of an object (this is covered in detail in Chapter 3); and release, a computation item (as discussed in the context of the $k$ cell above, and represented using sort ComputationItem) defined as part of the semantics for concurrency in KOOL.
that, when found in the current computation (in $k$), indicates that a lock is to be released.

**Equations and Rules in RLS**

Using the signature discussed above, a number of equations and rules are given to define the semantics of a language. In general, equations are used to define deterministic features, with rules defining nondeterministic and concurrent features.

Figure 2.8 shows several examples of equations used to define the semantics of KOOL, along with the definitions of the auxiliary operations used in the equations. The first provides the semantics for the statement $E ;$ (the stmt operation allows statements to be treated as part of the computation), saying that this is defined as the result of evaluating the expression $E$ and then discarding the result. Note that $\text{exp}(E)$ is placed “on top of” (i.e., to the left of) discard in the computation, meaning that it will be evaluated first, with the expectation that it will produce a value. The second equation provides a semantics for an expression made up of just the name $X$: $X$ is looked up to retrieve its current

\[ \text{op iv} : \text{Int} \rightarrow \text{Value} \ . \]
\[ \text{op } [\_\,,\_] : \text{Name Env} \rightarrow \text{ObjEnv} \ . \]
\[ \text{op release} : \rightarrow \text{ComputationItem} \ . \]

Figure 2.7: RLS Semantic Entity Definitions
value. The third equation provides the semantics for assignment: when assigning $E$ to $X$, $E$ is evaluated, and the resulting value is assigned to $X$ using the `assignTo` computation item.

One important point to note with these first three equations is that none of them mention the $k$ configuration item explicitly: these equations are valid anywhere the underlying constructs are encountered (note that in the third equation the semantics state that the value of $E$ will be assigned to $X$, but the assignment is done later, after $E$ is evaluated). Other equations, which define the semantics of constructs that depend on the current configuration, explicitly mention $k$ to ensure that they only apply when the defined construct is the next task to evaluate in the computation. This can be seen in the fourth equation, which defines the semantics of `lookup`. The environment, $Env$, is a finite map; $Env[X]$ is the lookup operation on the map, which should yield a location $L$ in the store where the value assigned to $X$ is held. The condition does this lookup, binding the location to $L$. This is represented using the $:=$ syntax, which binds the term, here just a variable, on the left hand side to the result of reducing the right hand side to a normal form, i.e., one where no rules or equations can apply. The equation then checks to see if $L$ is undefined, representing the case when a name not in the environment is used. If the location is defined, the value at the location is looked up in the store using the location lookup ($llookup$) computation item. This equation should only apply when the lookup is the next item in the computation to ensure that changes to the state are properly sequenced, ensuring here that the environment used is the currently active environment. One more point to note about this equation is the use of $CS$, which represents the other contents of configuration item `control`, a multiset of control-flow related items (the current computation, information about exceptions and loops, etc). A way to eliminate the need to mention

\[
\begin{align*}
\text{op stmt} : \text{Stmt} & \rightarrow \text{ComputationItem} . \\
\text{op exp} : \text{Exp} & \rightarrow \text{ComputationItem} . \\
\text{op discard} : & \rightarrow \text{ComputationItem} . \\
\text{op lookup} : \text{Name} & \rightarrow \text{ComputationItem} . \\
\text{op assignTo} : \text{Name} & \rightarrow \text{ComputationItem} . \\
\text{op llookup} : \text{Location} & \rightarrow \text{ComputationItem} . \\
\text{eq stmt}(E ;) = \text{exp}(E) & \rightarrow \text{discard} . \\
\text{eq exp}(X) = \text{lookup}(X) . \\
\text{eq stmt}(X < - E ;) = \text{exp}(E) & \rightarrow \text{assignTo}(X) . \\
\text{ceq control}(k(\text{lookup}(X) \rightarrow K) \ CS) \ env(Env) = \\
& \text{control}(k(\text{llookup}(L) \rightarrow K) \ CS) \ env(Env) \\
\text{if } L := Env[X] /\ \text{L }=\neq \text{undefined} .
\end{align*}
\]

Figure 2.8: RLS Semantics with Equations
\textbf{Figure 2.9: RLS Semantics with Rules}

such “unused” parts of the configuration added only for matching is part of \( K \), discussed in Section 2.5.

\textbf{Figure 2.9} shows the semantics of two concurrency-related features, defined using rules. The first rule, which is not conditional, defines the semantics of label statements, which provide a way for the user to give labels in program code that can then be used when model checking programs. When a label statement with label \( X \) is the next item in the computation, the label associated with the current thread is changed from \( X' \) (the former value) to \( X \). Here \( CS \), \( TS \), and \( KS \) are all used to represent unreferenced parts of the configuration. Configuration item \( t \) is a multiset with information for one thread, while \( \text{threads} \) is a multiset containing all the threads active in a program.

The second rule, which is conditional, defines the semantics for lock acquisition, given in \( K \) syntax in Chapter 3 Rule 3.33. Like Java, KOOL acquires locks on specific values. Here, a lock is being acquired on a value \( V \). The locks the thread currently holds are in the \( \text{holds} \) item (LTS), while the locks held by all threads are part of the \( \text{busy} \) item (LS). If the lock is not in LS (meaning it is not held by another thread), it can be acquired, which results in it being added to the \( \text{busy} \) item and to the thread-local \( \text{holds} \) item. When added to \( \text{holds} \) a lock count is also maintained, which is used to model cases where the same thread acquires multiple locks on the same value, ensuring that the locked value is released the proper number of times before it is removed from \( \text{busy} \) and can be acquired by another thread.

\section{2.5 \( K \)}

\( K \) \cite{155}, based on rewriting logic and the work on the computation-based style of rewriting logic semantics discussed above, is a general technique and notation for defining deterministic, nondeterministic, and concurrent computation. In this thesis, the focus is specifically on formal definitions of programming languages, which was the first application of \( K \). Beyond the prior work on rewriting logic semantics, \( K \) was also influenced by work on abstract state machines (ASMs) \cite{67}, the chemical abstract machine (CHAM) \cite{57}, and continuations \cite{171}. \( K \) takes its name from \( k \), the name of the configuration item used to hold the
current computation. While there are many similarities between K and the
computation-based style of rewriting logic semantics, there are some significant
differences as well, mainly in providing additional support for modularity and
for writing concise language definitions.

2.5.1 K Configurations

Configurations in K are defined identically to how they are defined using RLS.
In K, each configuration item is referred to as a K cell. The current computation
is still stored inside a k configuration item, or k cell, and it is still possible (as in
the case of threads) to have multiple copies of all cells, including k, if needed
by the semantics. A second standard cell, ⊤, represents the entire configuration
(i.e., the entire term).

2.5.2 K Sorts and Operations

Signatures in K are identical to signatures in RLS with one exception: several
new attributes for operations have been added, used by K to automatically
handle some routine language definition tasks.

The most common of these is strict, which is used on operator definitions
to indicate that the operands must be evaluated first before evaluating the entire
operation. This was done manually before in RLS, leading to a large number of
equations and operators used just to indicate how operands were being evaluated.
A common case is with arithmetic operations, such as addition, where the RLS
definition would be:

```
op _+_ : Exp Exp -> Exp .
op plus : -> ComputationItem .

eq exp(E + E') = exp(E, E') -> plus .
eq val(iv(I), iv(I')) -> plus -> K = val(iv(I + I')) -> K .
```

Here, two operators had to be defined. The first is the abstract syntax for
plus, which would be needed regardless; the second is a placeholder computation
item, added into the computation to indicate that the computation is “waiting”
for the two operands to be evaluated before evaluating the plus. The actual
equations are then shown. The first says that, to evaluate E + E’, one must first
evaluate E and E’, again using the plus computation item to indicate that an
addition will occur once E and E’ are evaluated. The second equation applies
after E and E’ have been evaluated to two integer values, I and I’. In this case,
the value returned is the sum of I and I’.

In K, this can be indicated as:

```

iv(I) + iv(I’) => iv(I + I’).
```
Behind the scenes, K will generate the intermediate operators to evaluate $E$ and $E'$ automatically, putting the values back into the positions of the original operands. This allows the semantic equation to use a form closer to the original syntax, instead of having the result values on top of an intermediate computation item in the computation. In cases where not every argument position should be strict, a list of natural numbers, indicating the strict positions, can also be provided. This is the case with a conditional, for instance, where one should evaluate the condition before choosing which branch is evaluated. For cases where the evaluation order is important, a variant of strict, seqstrict, can be used instead, which will enforce a left to right order of evaluation on all strict argument positions. Finally, note that the semantics given above use a rule (indicated as in rewriting logic with $\Rightarrow$), not an equation as may be expected; the reason for this is explained next.

### 2.5.3 K Rules and Equations

A K definition consists of two types of sentences: structural equations and rewrite rules. Structural equations carry no computational meaning, and, like equational logic equations, can be used for reasoning both from left to right and from right to left. When converted into term rewrite rules they are treated the same as equations in equational logic, evaluating from left to right. One use of structural equations is to provide definitions for auxiliary operations used in the semantics. Examples include operations to work with the lists and sets (list length, set membership, etc) included in the configuration, or operations to pull apart the abstract syntax to get useful information (the type of a declaration, the number of pointer “levels” in a C pointer declaration, the branches of a conditional, etc). Equations used to desugar language syntax are also considered to be structural, and include transformations such as turning a one-armed conditional into a two-armed conditional with a default else body.

One special type of structural equation, used with the strict and seqstrict attributes discussed above, is a heating/cooling rule. In the Chemical Abstract Machine, computations are represented as molecular soups, with information stored inside individual molecules. To allow computation, the information inside each molecule needs to move outside the molecule membrane, where it can encounter other information and interact. This process is called heating, and in K is represented by placing operands on top of the computation. In the Chemical Abstract Machine, the computation then cools, with the new compounds (i.e., the results of the computation) going back into molecules where they can no longer directly interact. The K equivalent is when the computed values are placed back into the original abstract syntax item, like $iv(I) + iv(I')$ above.

While it is not necessary to write these rules by hand – one can assume that they are automatically created by the use of strictness attributes – it is possible to do so. When written manually, they are given a special notation using the $\Leftarrow$.
symbol to separate the two sides of the equation. This is solely to provide added intuition, and could be represented using two equations instead, one using the unevaluated form (like \(a_1\)) and one using the evaluated form (like \(i_1\)). Examples of heating and cooling rules include:

\[ a_1 + a_2 \rightleftharpoons a_1 \bowtie + a_2 \quad (2.2) \]

\[ i_1 + a_2 \rightleftharpoons a_2 \bowtie i_1 + \square \quad (2.3) \]

\[ \text{if } b \text{ then } s_1 \text{ else } s_2 \rightleftharpoons b \bowtie \text{ if } \square \text{ then } s_1 \text{ else } s_2 \quad (2.4) \]

K automatically generates special operators, serving the same purpose as the \texttt{plus} computation item defined manually above, to represent the intermediate steps being taken in the computation. When an operand is placed on top of the computation through heating, the operand position is replaced with a \(\square\), leading to operators like \(\square + \square\) in equation 2.2, \(+ \square\) in equation 2.3, and \(\text{if } \square \text{ then } \text{else }\) in equation 2.4. The cooled value would then go back into the position of the box. Note that the equations in 2.2 and 2.3 show the deterministic version (\texttt{seqstrict}), since the first operand must evaluate to a value before the second is evaluated.

Unlike structural equations, rewrite rules represent actual steps of computation. Examples include:

\[ i_1 + i_2 \rightarrow i, \text{ where } i \text{ is the sum of } i_1 \text{ and } i_2 \quad (2.5) \]

\[ \text{if } \text{true } \text{then } s_1 \text{ else } s_2 \rightarrow s_1 \quad (2.6) \]

\[ \text{if } \text{false} \text{ then } s_1 \text{ else } s_2 \rightarrow s_2 \quad (2.7) \]

Rule 2.5 is the standard addition rule, taking \(i_1 + i_2\) to the sum of \(i_1\) and \(i_2\). Like rules in rewriting logic, K rules represent a one-way transition, allowing reasoning from left to right only. Rules 2.6 and 2.7 provide the semantics for the if statement, with the correct path chosen based on whether the condition evaluates to \texttt{true} or \texttt{false}.

Up to now, the rules have not referenced K cells. The cells are given in K rules using an XML-like notation, with an opening cell “tag”, like \(\langle k \rangle\) and a closing tag like \(\langle /k \rangle\). The last rule, rule 2.8, shows an example using multiple cells with this XML-like notation. This is a variant of the KOOL assignment rule, with variable \(X\) being assigned value \(V\). The environment (\texttt{env}) and store (\texttt{mem}) cells both hold finite maps, represented as a set of pairs, with operations defined for both to ensure the uniqueness of the first projection of each pair. As in Figure 2.9, \(TS\) and \(CS\) are used to match other parts of the thread and
control states:

\[
\langle t \rangle \langle \text{control} \rangle \langle k \rangle X \leftarrow V \bowtie K \langle /k \rangle CS \langle /\text{control} \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle TS \langle /t \rangle
\]

\[
\langle \text{mem} \rangle (L, V') Mem \langle /\text{mem} \rangle \rightarrow
\]

\[
\langle t \rangle \langle \text{control} \rangle \langle k \rangle K \langle /k \rangle CS \langle /\text{control} \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle TS \langle /t \rangle
\]

\[
\langle \text{mem} \rangle (L, V) Mem \langle /\text{mem} \rangle \quad (2.8)
\]

At this point, Rule 2.8 looks like an RLS rule, but with the cells given in an XML-like notation instead of prefix notation. K includes special notation to help simplify rules and make them more modular. The most important is that context needed only for matching the configuration structure, including variables such as CS and TS and cells such as t and control, can be elided:

\[
\langle k \rangle X \leftarrow V \bowtie K \langle /k \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle
\]

\[
\langle \text{mem} \rangle (L, V') Mem \langle /\text{mem} \rangle \rightarrow
\]

\[
\langle k \rangle K \langle /k \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle
\]

\[
\langle \text{mem} \rangle (L, V) Mem \langle /\text{mem} \rangle \quad (2.9)
\]

This is not just a notational convenience: requiring this extra context makes the rules less modular, since changes to the layout of the configuration (adding a new level, moving cells between levels) would require changes to the rule. Since this information is still needed for matching, it is added back in using context transformers, which will transform each rule into a rule with a complete matching context, based on the structure of the configuration. Context transformers are discussed further below.

Another feature, which is just a notational convenience, is the ability to replace variables that are given on the left-hand side of a rule but are not otherwise used (in conditions or on the right-hand side) with an underscore, similar to functional languages such as OCaml [153, 4]. This is used below to replace V', given in Rule 2.9 with an underscore, since it is not used elsewhere in the rule:

\[
\langle k \rangle X \leftarrow V \bowtie K \langle /k \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle
\]

\[
\langle \text{mem} \rangle (L, \_ ) Mem \langle /\text{mem} \rangle \rightarrow
\]

\[
\langle k \rangle K \langle /k \rangle \langle \text{env} \rangle (X, L) Env \langle /\text{env} \rangle
\]

\[
\langle \text{mem} \rangle (L, V) Mem \langle /\text{mem} \rangle \quad (2.10)
\]

Since matching against lists and sets is used quite often, it is also helpful to have special notation for both lists and sets. In K, this is indicated by using “...”, with a “...” at the start or end of a cell indicating a list match (“...” at the start would indicate that one is matching the tail of the list, while “...” at the
end would indicate that one is matching the head), and “...” at both ends of the cell indicating a set or multiset match. Rule 2.10 can be transformed to use this notation, with the result given below in Rule 2.11:

\[
\langle k \rangle X \leftarrow V \ldots \langle /k \rangle \quad \langle \text{env} \rangle \ldots (X,L) \ldots \langle /\text{env} \rangle \\
\langle \text{mem} \rangle \ldots (L, \_ ) \ldots \langle /\text{mem} \rangle \rightarrow \\
\langle k \rangle \cdot \ldots \langle /k \rangle \quad \langle \text{env} \rangle \ldots (X,L) \ldots \langle /\text{env} \rangle \\
\langle \text{mem} \rangle \ldots (L,V) \ldots \langle /\text{mem} \rangle 
\]

(2.11)

In Rule 2.11 the “...” convention allows us to just mention the pair being used in the environment and store, while eliding the rest of the cell contents. \( k \) is a list, so the convention indicates that we are just matching the head. Since we are eliminating the head element, and not replacing it with anything else, we indicate this with \( \cdot \).

Finally, some of the information in Rule 2.11 is redundant. For instance, the environment does not change, but must be mentioned on each side of the rule, since mentioning it just on the left-hand side would be the same as discarding it. K provides a way to indicate that only certain parts of a term are changed by a rule to reduce this redundancy. To do this, in the K notation it is possible to underline the parts of a term that change, and then note the change (i.e., the right-hand side) underneath the line (i.e., under the left-hand side). The arrow between the two sides (\( \rightarrow \)) is also dropped, since it is no longer needed; to reconstruct the right-hand side of the rule, the “top” of the K rule can be copied to the right-hand side, with any underlined portions replaced by the subterms given below the line.

\[
\langle k \rangle X \leftarrow V \ldots \langle /k \rangle \langle \text{env} \rangle \ldots (X,L) \ldots \langle /\text{env} \rangle \langle \text{mem} \rangle \ldots (L, \_ ) \ldots \langle /\text{mem} \rangle \\
\_ . V
\]

(2.12)

Rule 2.12 shows an example where this has been done. In the \( k \) cell, the \( \cdot \) is now under \( X := V \), while \( V \) is now under \( \_ \). Since both the environment and the first element \( (L) \) of the pair in \( \text{mem} \) do not change, each only needs to be written once.

### 2.5.4 Rules and Equations in K and RLS

The use of rules and equations is slightly different in K and RLS. In RLS, equations are used to represent deterministic language features, while rules represent language features that can be affected by nondeterminism and concurrency.

---

5This is purely a notational convention, to indicate that the match is being performed modulo both associativity and commutativity; it would be possible to change the notation to just use one “...”, but this would make the type of match less obvious in the rule.
such as memory reads and writes that could lead to data races, or attempts to acquire locks. Some recent work [121] has instead looked at the use of rules and equations in terms of computational abstraction, with the ability to move from a specification with all equations to one with all rules seen as turning an “abstraction knob” from a more abstract (all equations) to less abstract (all rules) definition. This allows the language designer to capture the idea of a computational step more closely, with each rule representing a discreet step (like a transition in a structural operational semantics definition).

This latter view is the perspective taken in K. Computational steps are defined as rewrite rules, while other parts of the computation are defined using structural equations, with the ability to “tune” this based on what should be considered a step of computation. In mapping K to Maude, structural equations and rules can be mapped directly to Maude equations and rules, although this mapping will often be modified based on the planned use of the semantics. For instance, for model checking, most K rules will be mapped to equations, with only those rules directly interacting with concurrency mapped to Maude rules.

### 2.5.5 Context Transformers

To ensure that rules are modular, it should be possible to continue using a rule, unchanged, when parts of the context (i.e., K cells) not mentioned in the rule are modified or replaced. Given a specific rule, the easiest case to deal with is when the part of the configuration matched by a rule remains the same but the surrounding context changes. This case is handled naturally in both RLS and K by defining the configuration as an associative, commutative multiset of configuration items (often referred to as a “soup”). Because the configuration is commutative, the ordering of the individual items does not matter, meaning it is not essential to put items in positions and then name them even when not used, as it would be if the configuration were represented as a tuple. Associativity then allows arbitrary groupings of items, meaning we don’t have to worry about the order in which the set is formed (i.e., there is no difference between set 1 (2 3) and set (1 2) 3, both can just be treated as 1 2 3).

The more challenging case is when the configuration used by a rule is changed. A common example is when new levels are added to the cell hierarchy. For instance, when threads are added to a language, thread-specific information is grouped into a thread cell, often called $t$, while information that is global to the computation is left at its current level. Rules that referenced both information that is now inside the thread cell and global information would then need to be changed to account for this added level of nesting. An example is Rule 2.8 the assignment rule, which used the computation ($k$), environment ($env$), and store ($mem$) cells. When threads are added, the computation and environment are grouped inside $t$, but the store is not, since there is only one which is shared by all threads. In RLS this leads to a revision of the assignment rule, shown
below in Rule 2.13

\[ \langle t \rangle \ldots \langle k \rangle X := V \ldots \langle /k \rangle \langle \text{env} \rangle \ldots \langle X, L \rangle \ldots \langle /\text{env} \rangle \ldots \langle /t \rangle \langle \text{mem} \rangle \ldots \langle L, V \rangle \ldots \langle /\text{mem} \rangle \quad (2.13) \]

Rule 2.13 shows the type of change typically made within a language, but it is also important that language features, once specified, can be used across languages, providing a method to build languages more quickly by assembling them from trusted pieces. This provides an additional challenge, since the configuration in a different language may be quite different from that used in the language where the feature was originally defined. In this case, using RLS rules would often need to be manually changed from one language to another to accommodate differences in configurations. K solves this problem through the use of context transformers:

**Definition 4** A context transformer, \( t \), is a function which takes as input a K configuration \( k \) and a K sentence (i.e., a K rule or equation) \( s \) and generates a new sentence, \( s' \). For any two cells \( c \) and \( c' \) in \( s \), \( s' \) is created by \( t \) to include any cells \( c_n \) which occur between \( c \) and \( c' \) in \( k \). This process, context completion, allows a correct match against the configuration structure \( k \) using \( s' \). In cases where \( t \) is unable to calculate a unique completion for \( s' \), \( s \) is said to be ambiguous.

Context completion works using a combination of the information given in the configuration (such as indications that cells can be repeated), information in \( s \) (any already specified context), and various rules and heuristics (such as using the shortest path in \( k \) between \( c \) and \( c' \)). Since in general there are many possible completions, the rules and heuristics are needed to identify a single completion, which should be the intended completion (i.e., the completion containing the context that would otherwise have been written by hand). If \( s \) is ambiguous, this means that there are multiple possible completions that comport with the given rules and heuristics, so additional context information must be added manually to \( s \) before \( s' \) can be generated by \( t \).

Using context transformers, only those portions of the configuration actually used in a rule need to be mentioned, with other parts of the configuration, needed only to ensure a valid match, added automatically. For instance, the assignment rule shown originally in Rule 2.8 can remain as is, without the need to explicitly add the thread cell as was done for Rule 2.12. This accommodates changes within a language and changes between languages, under the assumption that the cell names themselves do not change.

### 2.5.6 K Notation Changes

Since it was first introduced [78, 30] the K notation has gone through several iterations, mainly involved with how the cells are represented. At first, K cells were represented using standard Maude notation – the computation cell would
be represented as \( k(K) \), with \( K \) being the actual computation inside the cell. This later changed to allow for specific notation for lists and sets inside cells, replacing the standard opening or closing parentheses with a (left or right) angle to represent that more information was included “in that direction”, represented now using “...”. For instance, to match the first element in the computation, the notation would use \( k(\text{item} \triangleright \triangleright) \), adapted later to \( k(\mid \text{item} \uparrow) \). A more mathematical notation was then adopted, with the cell label added as a subscript to the cell, like \( \langle \text{item} \rangle_k \). Based on feedback from others, the current notation uses the XML-like notation presented in this thesis. An attempt has been made to ensure all semantic rules presented in this thesis that are written in K, and not Maude, syntax make use of this current notation, but this change needs to be kept in mind when referencing earlier work.
Language design is much more an art than a science. Selecting from the wide range of available language features, designing new features, and choosing appropriate syntax are all important tasks which do not have clear "best" answers. Sometimes even small decisions in any of these areas can drastically impact the usability, or feel, of a language.

One method to improve the language design process is language prototyping. Prototypes provide the same advantages for designing languages as they provide for designing programs: by actually using the language features being designed, instead of just seeing them on paper, the designer can gain confidence that the features work well, or discover early in the process that they do not. Also, definitions of features in a prototype must include enough detail to allow them to be used with programs, highlighting areas of the design that appear complete on paper but are unclear in practice. The ability to use the prototyped features also provides opportunities to see how different features of the same language interact, helping to prevent poor feature interactions from making it into a settled version of the language. Having a method for prototyping languages that does not require the time and effort involved in modifying a compiler or interpreter provides additional benefits, allowing the design process to proceed more rapidly.

Ideally, language prototypes will be formal, providing a clearer definition of the features being designed than is possible with normal textual descriptions, such as those found in manuals. Having a formal definition also opens the process up to the use of formal tools, such as model checkers, state space search tools, and theorem provers. These not only allow proofs about programs and properties of the language (the latter referred to as language meta-theory proofs), but can also provide ways to gain confidence that a language is working as expected. For instance, using state space search techniques, it may be possible to show that only expected states can be reached in certain sample programs when using certain concurrency features in a language. This type of tool support is especially important with the addition of complex abstraction capabilities and new concurrency features, both of which make programs and programming languages more challenging to understand.

The remainder of this chapter illustrates language prototyping using a case
study of the KOOL language. KOOL is a class-based, dynamic object-oriented language, supporting those features commonly understood to make a language object-oriented: encapsulation, inheritance, and polymorphism. Section 3.1 provides an introduction to the language, with several examples illustrating standard language features. Section 3.2 documents the abstract syntax of KOOL, while Section 3.3 then provides a high-level overview of the KOOL state configuration and the algebraic representation of KOOL program values (such as objects and object references). Next, Section 3.4 describes the base semantics of KOOL, including semantic rules for object creation, method dispatch, super and self references, and exceptions.

To illustrate language prototyping, Section 3.5 details the first major extension of KOOL, the introduction of a concurrency model similar to that used in Java, with multiple threads and locks acquired on objects. Two additional extensions are then highlighted in Section 3.6. Details of the implementation not covered in the other sections in this chapter are discussed in Section 3.7.

### 3.1 Introducing KOOL

KOOL is dynamic, class-based object-oriented language, loosely inspired by, but not identical to, the Smalltalk language [64]. KOOL includes support for standard imperative features, such as assignment, conditionals, and loops with break and continue. Many familiar object-oriented features are also supported:

- all language values are represented by objects;
- all operations are carried out via message sends;
- inheritance is based around a standard single inheritance model, with a designated root class named `Object`;
- all method calls use dynamic dispatch, with the appropriate method to invoke selected based on the dynamic class of the target object;
- methods are all public, while fields are all private outside of the owning object;
- scoping is static, yet declaration order for classes and methods is unimportant.

In addition, KOOL allows for the run-time inspection of object types via a `typecase` construct, similar to a standard case construct but branching on types instead of program values, and includes support for exceptions with a standard `try/catch` (no `finally`) mechanism.

The concrete syntax of KOOL is shown in Figure 3.1. The lexical definitions of literals are not included in the figure to limit clutter, but are standard (for instance, booleans include both `true` and `false`, strings are surrounded with
**Program**  
\[ P ::= C^* E \]

**Class**  
\[ C ::= \text{class } X \text{ is } D^* M^* \text{ end } \mid \text{class } X \text{ extends } X' \text{ is } D^* M^* \text{ end} \]

**Decl**  
\[ D ::= \text{var } \{X\}^+ ; \]

**Method**  
\[ M ::= \text{method } X \text{ is } D^* S \text{ end } \mid \text{method } X(\{X'\}^+) \text{ is } D^* S \text{ end} \]

**Expression**  
\[ E ::= X \mid I \mid F \mid B \mid Ch \mid Str \mid (E) \mid \text{new } X \mid \text{new } X(\{E\}^+) \mid \text{self} \mid E X op E' \mid E.() \mid \text{super}() \mid \text{super}X(()) \mid \text{super}(\{E\}^+) \mid \text{primInvoke}(\{E\}^+) \]

**Statement**  
\[ S ::= E \leftarrow E' ; \mid \begin{cases} D^* S \text{ end } & \text{if } E \text{ then } S \text{ (else } S' \text{)} \end{cases} \text{fi} \mid \text{try } S \text{ catch } X S \text{ end} \mid \text{throw } E ; \mid \text{while } E \text{ do } S \text{ od} \mid \text{for } X \leftarrow E \text{ to } E' \text{ do } S \text{ od} \mid \text{break} ; \mid \text{continue} ; \mid \text{return } (E) ; \mid S S' \mid E ; \mid \text{typecase } E \text{ of } C s^* \text{ (else } S \text{)} \end{cases} \text{end} \]

**Case**  
\[ Cs ::= \text{case } X \text{ of } S \]

\[ X \in \text{Name}, I \in \text{Integer}, F \in \text{Float}, B \in \text{Boolean}, Ch \in \text{Char}, Str \in \text{String}, X op \in \text{Operator Names} \]

---

Figure 3.1: KOOL Syntax

double quotes, etc). Most message sends are specified in a Java-like syntax; those representing binary operations can also be used infix (\(a + b\) desugars to \(a.+(b)\)), with these infix usages all having the same precedence and associativity. Finally, semicolons are used as statement terminators, not separators, and are only needed where the end of a statement may be ambiguous – at the end of an assignment, for instance, or at the end of each statement inside a branch of a conditional, but not at the end of the conditional itself, which ends with \(\text{fi}\).

One important point is that many standard expressions are not defined as part of the KOOL syntax, but instead are handled using message sends, such as the arithmetic, logical, and boolean operations commonly found in programming languages.

Figure 3.2 shows an example of the typical “Hello World!” program in KOOL. A program in KOOL is made up of any (0 or more) user defined classes, followed by an arbitrary KOOL expression, which has access to these classes as well as classes provided in a standard KOOL prelude. \texttt{console} is a predefined object, representing the standard input and output streams, with methods to read from (\(\gg\)) and write to (\(\ll\)) the stream\(^1\). "Hello World!" is a string, which is actually an object of class \texttt{String}. When \(\ll\) is invoked with an argument, it will call that object’s \texttt{toString} method and then return itself, allowing multiple calls to \(\ll\).

\(^1\)These method names were borrowed from C++.

```
console << "Hello World!"
```

Figure 3.2: Hello World in KOOL

---

---
class Factorial is
  method Fact(n) is
    if n = 0 then return 1;
    else return n * self.Fact(n-1);
  fi
end
end

console << (new Factorial).Fact(200)

Figure 3.3: Recursive factorial in KOOL

to be chained (such as console << "Value = " << 3). Since calling toString on a string will just return the string value, this program just writes "Hello World!" to standard output.

Figure 3.3 shows a slightly more complex example. Here, a new class Factorial is defined with a method Fact that calculates the factorial of the parameter n. All the operations are actually method calls, including the equality test in the condition, the multiplication of n and self.Fact(n-1), and the subtraction of 1 from n. This means that the operators used (such as *) are defined as method names, not as reserved words or keywords in the language, providing a more uniform syntax and giving users a mechanism to define their own operators. The conditional expects an object of class Boolean to be given as the guard, but otherwise imposes no restrictions on types. Finally, methods in KOOL can be invoked recursively, with self here indicating that the current object is also the target of the message send (a target always needs to be indicated, even if it is self). The program expression uses class Factorial by creating a new instance of the class, using new, and then invoking its Fact method by passing in a number. The result is then printed to the console.

Finally, Figure 3.4 presents two sample class definitions, providing a simple example of inheritance and calls to super-methods using a familiar Point and ColorPoint example. Class Point represents a point in 2D space, with x and y coordinates, and implicitly inherits from (extends) class Object. Class ColorPoint explicitly extends class Point, adding a new variable c to represent the color of the point. The constructor for ColorPoint calls its parent constructor, passing the x and y coordinates, while the version of toString defined in ColorPoint also uses super, here to invoke the parent version of the toString method. + is defined in the String class as string concatenation.

Additional examples of KOOL programs are available with the KOOL distribution, available on the KOOL website [86].
class Point is
  var x, y;

  method Point(in_x, in_y) is
    x <- in_x;
    y <- in_y;
  end

  method toString is
    return ("x = " + x.toString() + " and y = " + y.toString());
  end
end

class ColorPoint extends Point is
  var c;

  method ColorPoint(in_x, in_y, in_c) is
    super(in_x, in_y);
    c <- in_c;
  end

  method toString is
    return (super.toString() + " and c = " + c.toString());
  end

  method write is
    console << self;
  end
end

(new ColorPoint(20, 30, 5)).write

x = 20 and y = 30 and c = 5

Figure 3.4: Inheritance and Built-ins in KOOL

3.2 Abstract Syntax

The abstract syntax of KOOL closely mirrors the concrete syntax, but in general does not require as many cases, since in some cases several productions in the concrete rules can be mapped to a single abstract syntax production. For instance, binary infix operators are mapped to standard “dot-notation” message sends. The definitions of abstract syntax shown below are provided using standard Maude notation.

Names: The abstract syntax for names in KOOL takes advantage of the Maude QID module, providing a way to represent arbitrary identifier strings as quoted (i.e., preceded by a single quote, such as ’Boolean) strings in the semantics. These quoted strings are wrapped in a special operator n, injecting them into the Name sort. Lists of names, needed for parameter lists in method definitions and in variable declarations, are also provided, with names in the list separated by commas.

fmod NAME-SYNTAX is
  protecting QID .

  sort Name NameList .
  subsort Name < NameList .
Scalars: Scalars to represent integers, floating-point numbers, booleans, and individual characters are also provided in the KOOL abstract syntax. Each scalar is provided with a K-specific sort (e.g., KInt) into which it is injected. The underlying representations are those offered by Maude. Strings are not strictly scalars, but are literals, and so are included here as well for convenience.

Expressions: Expressions in KOOL include (through subsorting) the prior definitions of names and scalars. The abstract syntax for expressions also adds expression lists, used in method calls, and a special expression, Nil, which represents an object reference assigned to no object.

Statements: The base sort for statements in KOOL is Stmt. An expression can be used as a statement by terminating it with a semicolon, represented with the ; operator definition. A skip; statement is also introduced, representing a statement that does nothing.
Declarations: In the KOOL abstract syntax, declarations of class fields and local variables in methods use the \texttt{var} keyword followed by a list of names.

Sequential Composition: Sequential composition is represented by having two statement abut. No separator is needed since statements are terminated either by keywords (\texttt{fi} for if statements, for instance) or by semicolons.

Blocks: Blocks, which provide lexical scoping, are opened with the \texttt{begin} keyword and closed with the \texttt{end} keyword. The block includes a list of declarations followed by a list of statements. Each is optional in the concrete syntax, with default values provided in the abstract syntax to allow a normalized form of the construct. Note that requiring declarations to come before statements in the block, versus mixing the two, is done just to simplify parsing; the declaration semantics add the name into the environment when its declaration is encountered, meaning a declaration given in the middle of a sequence of statements would only be visible after the declaration is given, not before, which would ensure proper scoping.
**Assignments:** Assignment uses <- notation, with the value on the right and the target on the left.

```fmod
 ASSIGNMENT-SYNTAX is
   including EXP-SYNTAX .
   including STMT-SYNTAX .

   op _.<-_; : Exp Exp -> Stmt .
endfm
```

**Conditionals:** Conditionals, with and without else branches, are supported in the abstract syntax. Only one is used in the semantics: a conditional without an else clause is transformed into a conditional with an else skip; clause. Note that this could also be done on the translation from concrete to abstract syntax.

```fmod
 CONDITIONAL-SYNTAX is
   including EXP-SYNTAX .
   including STMT-SYNTAX .

   op if_then_else_fi : Exp Stmt Stmt -> Stmt .
   op if_then_fi : Exp Stmt -> Stmt .
endfm
```

**Loops:** for, while, and do/while loops are all supported in KOOL. KOOL also allows break and continue statements, which either exit or restart (in the case of the for loop, after incrementing the loop counter) the current loop.

```fmod
 LOOP-SYNTAX is
   including EXP-SYNTAX .
   including STMT-SYNTAX .
   including NAME-SYNTAX .

   op for_<-_-to_do_od : Name Exp Exp Stmt -> Stmt .
   op while_do_od : Exp Stmt -> Stmt .
   op do_while_od : Stmt Exp -> Stmt .
   op break'; : -> Stmt .
   op continue'; : -> Stmt .
endfm
```

**Methods:** A method in KOOL uses the method keyword, followed by the method name, and then a list of method parameters. The method body is bracketed with is and end, and includes a list of local declarations (optional in the concrete syntax) and the method statement. The abstract syntax treats multiple methods as a list, but these are converted into a method set in the internal representation, since the order of declaration does not matter. Defined with the method syntax is the abstract syntax for return. A return statement without an expression is equivalent to returning the nil value.

```fmod
 METHOD-SYNTAX is
   including NAME-SYNTAX .
```
including DECL-SYNTAX.
including STMT-SYNTAX.

sorts Method MethodList.
subsort Method < MethodList.

op empty : -> MethodList.
op _,_ : MethodList MethodList -> MethodList [assoc id: empty].
op method_'(_')is__end : Name NameList Decls Stmt -> Method.
op return'; : -> Stmt.
op return_; : Exp -> Stmt.
endfm

Classes: Classes in KOOL are given a class name, the name of the extended class (this is not required in the concrete syntax, but is automatically added in the abstract syntax – all classes that do not explicitly extend another class extend Object), declarations of class fields, and a list of class methods. primclass is used to identify that a class is used to model a primitive value in the language, which enables support for manipulating this primitive value using the KOOL primitives functionality.

fmod CLASS-SYNTAX is
  including DECL-SYNTAX.
  including NAME-SYNTAX.
  including METHOD-SYNTAX.

  sorts Class Classes.
  subsort Class < Classes.

  op _,_ : Classes Classes -> Classes [assoc id: empty].
op empty : -> Classes.
op class_extends_is__end : Name Name Decls MethodList -> Class.
op primclass_extends_is__end : Name Name Decls MethodList -> Class.
endfm

Self: The self keyword represents the current object. It can be used wherever a reference to the current object is needed, and also must be used when making method calls where the current object is also the target.

fmod SELF-SYNTAX is
  including NAME-SYNTAX.
  including Exp-SYNTAX.

  op self : -> Exp.
endfm

Super: The super keyword is used when making calls back up the inheritance hierarchy. The abstract syntax supports two forms of super: the first is used in regular sends, where super replaces the target object, while the second is used inside constructors, in the case where an object wants to invoke a parent constructor.
Message Sends: Message sends are given in a Java-like syntax, with the target, followed by a dot, followed by the name of the method, with a list of expressions then given in parentheses to represent the arguments for the call. All message sends are normalized into this form during the translation from concrete syntax, including those that use infix operators (like +).

New: The abstract syntax for `new` accepts a name, which is the name of a class, and a list of expressions representing the arguments to the class constructor.

Exceptions: Exceptions use a standard `try/catch` syntax, similar to that used in Java. `try` includes the statements being handled by the exception handler. `catch` includes a name, used to represent the exception value when an exception is thrown, and a statement, which should handle the exception. Exceptions can be thrown directly by the language semantics, and can also be thrown using the `throw` keyword, which will throw the value yielded by the given expression.

Typecase: The `typecase` syntax provides a way to branch to different functionality based on the runtime type of an expression. The `typecase` keyword takes an expression, a list of cases, and an optional `else` case, used when no
other case matches. Each case accepts a name, which should be of a class, and the statement to execute when that case holds.

```plaintext
fmod TYPECASE-SYNTAX is
    including NAME-SYNTAX.
    including EXP-SYNTAX.
    including STMT-SYNTAX.

    sorts ElseCase Case Cases.
    subsort Case < Cases.

    op empty : -> Cases.
    op _,_ : Cases Cases -> Cases [assoc id: empty].
    op case_of_ : Name Stmt -> Case.
    op typecase_of_end : Exp Cases -> Stmt.
    op typecase_of__end : Exp Cases ElseCase -> Stmt.
    op else_ : Stmt -> ElseCase.
endfm
```

**Primitives:** The `primInvoke` keyword provides a way to invoke KOOL primitives within a method. The first parameter is the number of the primitive in the KOOL primitive map; the remaining parameters are the arguments to the primitive call.

```plaintext
fmod PRIMITIVES-SYNTAX is
    including EXP-SYNTAX.

    op primInvoke_ : ExpList -> Exp.
endfm
```

**Programs:** KOOL programs are made up of a list of classes and an expression. The abstract syntax uses the `__` operator, abutting the classes and main expression.

```plaintext
fmod PROGRAM-SYNTAX is
    including CLASS-SYNTAX.
    including EXP-SYNTAX.

    sort Program.

    op __ : Classes Exp -> Program.
endfm
```

### 3.3 State Infrastructure and Value Representations

One key design decision for a programming language is to determine the class of values which can be specified, manipulated, and stored by a language. Another, necessary for determining a formal definition of the language, is to determine
the structure of the language configuration. These two decisions form a basis for the language semantics, which will manipulate the language values and will use various parts of the configuration inside rules. Both value representations and parts of the KOOL configuration are described below; a description of the semantics is deferred until Section 3.4.

3.3.1 KOOL Value Representations

Figure 3.5 shows the values available in the KOOL language. Two of the values are used to represent object references: nil, which represents the case where there is no valid reference, such as with a variable that has not been assigned an object value; and oref, or “object reference”, which references a specific location in the KOOL memory representation. Note that oref is “opaque”: unlike pointers in C, it is not possible to “look inside” object references, create them by hand (such as casting a number to a reference), or deallocate them. This is similar to most object-oriented languages, such as Smalltalk and Java.

Objects are also represented as values, while the object itself is represented as a multiset of individual object “parts”, also just identified as having sort Object. This includes an object environment oenv, used to model field scope in inheritance and described in more detail in Section 3.4, and myclass, used to hold the name of the dynamic class of an object (i.e., the class used to actually create the object). The multiset formation operation, \text{...}, allows these items, or new items defined in language extensions, to be put together to create an object.

3.3.2 KOOL State Infrastructure

As discussed in Chapter 2, the state is specified as a multiset of K cells, with each cell holding information needed for the computation, and with the cells formed into a hierarchy. This hierarchy of cells forms the context used by the K rules that formally specify the semantics of the various language features being defined. The layout of the state is determined both by the needs of the language (as will be shown when KOOL is extended to support concurrency, it must be possible to group certain cells into threads) and by convenience, allowing cells to be organized in ways that make the definition more understandable.

The KOOL state is made up of multiple cells with a single explicit layer of nesting, grouping the components responsible for defining control flow together.
A visual depiction of the state is shown in Figure 3.6.

During program execution, KOOL tracks local names (such as from method parameters) that are in scope and their current memory locations. This is stored in env. These memory locations then map to values in mem, with the next free memory location in nextLoc. KOOL includes a garbage collector, described in Section 6.1.2. Input and output are stored in the input and output state components, respectively.

Those state components related directly to execution control are stored in control. This includes several stacks that are used to quickly recover the program to a state saved at a prior point in time: the method stack (mstack), exception stack (estack), and loop stack (lstack). While not strictly necessary, they save the effort of having to selectively unwind the control context to get back to the proper context for handling a method return or exception catch, for instance. Also included is the current computation, or $k$, which provides an explicit representation of the current stream of execution and also gives its name to our definitional approach. More details about the method and exception stacks are provided in Section 3.4.

Finally, included are several components needed just for the object-oriented features of the language. cobj contains a reference to the current object context – the object inside which execution is occurring. Since an object can act as either its dynamic class or one of its ancestor classes (for instance, in super calls, or in cases where a method is called that is defined in an ancestor and not overridden), cclass indicates the current class context as well. The class set ($cset$) contains the class definitions available in the program, including user-defined classes and classes specified in the prelude. The class definitions themselves contain information structured as sets, with items representing the class name, parent class name, and sets of methods, among others. Sets are used because of their flexibility; tuples would need to be changed if more information needs to be added to the tuple in an extension to the language, while sets do not: new class items can be added without invalidating the current rules, which only match those items they need.
### 3.4 Dynamic Semantics

As with any non-trivial language, there are actually a fair number of $\text{K}$-rules needed to give the semantics of the language. This section examines some of the more interesting features; the complete definition of the language is available online \[86\]. Note that KOOL was defined using an earlier version of $\text{K}$; the definition given here uses the newest version of the $\text{K}$ notation, but may not use all the newest features.

The semantics for each area of functionality are separated into individual figures. Of the operators that are used, most are left undefined, since the definition can be derived easily from the context in which the operator is used. For instance, an operator $\text{op}(X)$ takes a name as a parameter and, if it is on the computation, is a computation item. Thus, it has signature $\text{op} : \text{Name} \rightarrow \text{ComputationItem}$.

There are two exceptions to this. First, operators are defined for all syntactic constructs in the language in the figure in which they are used. This helps make the leap from the syntax of the language to the semantics. Second, operators are defined if they have attributes, since there would be no other way to know that they have the attributes they have been given. The rules make use of context transformers, described in Section 2.5.5, allowing us to leave out parts of the configuration used just to match across the hierarchy. All language syntax is presented in a \textit{sans serif} font, while semantics are presented in \textit{italics}.

#### 3.4.1 Common Operations

Figure 3.7 shows some $\text{K}$ definitions that are used in the definitions of other features, the first two being common to many of the languages defined as part of the RLS work so far in $\text{K}$. The first contextual rule, Rule (3.1), defines how the value $(V)$ corresponding to a location $(L)$ is retrieved from memory, when the lookup operation is the next task in the computation and $(L,V)$ is a pair contained in the set representing the memory.

Rule (3.2) has a two-hole context, one identifying the value-to-location-assign task on top of the computation and the other identifying the pair corresponding to the location in the store; once matched, the assign task is eliminated (hence the use of the identity “·”) and the current value at that location is replaced by the assigned value. Note the use of an underscore for the current value – similarly to many functional languages, no name is given to this value since it will not be referred to elsewhere.

The remaining $\text{K}$-rules in Figure 3.7 define several operators typical in OO programming language definitions, such as ones for locating the parent class, the fields, or a particular method of a class, or the set of names of classes inherited by a class; the syntax of these operators is defined at the bottom of Figure 3.7.

Rules (3.3) and (3.4) are self explanatory; information for each class is represented as a multiset of class items “wrapped” with the constructor $\text{cls}$,
\[
\langle k \rangle \text{lookup}(L) \ldots (/k) \langle \text{mem} \rangle \ldots (L, V) \ldots (/\text{mem})
\]

(3.1)

\[
\langle k \rangle V \leftarrow \text{assign}(L) \ldots (/k) \langle \text{mem} \rangle \ldots (L, \_ ) \ldots (/\text{mem})
\]

(3.2)

\[
\text{parent}(C, \text{cls}(\text{cname}(C)) \text{name}(C') \ldots)
\]

(3.3)

\[
\text{flds}(C, \text{cls}(\text{cname}(C)) \text{flds}(X)) \ldots)
\]

(3.4)

\[
\text{getInheritsSet}(\text{Object}, C, \ldots)
\]

(3.5)

\[
\text{getInheritsSet}(C, \text{parent}(C), C, C, \ldots) \text{[owsise]}
\]

(3.6)

\[
\text{getMthd}(X, C, (\text{cls}(\text{cname}(C)) (\text{mthd}(\text{name}(X)) MI: \text{MthdItms})) \ldots)
\]

(3.7)

\[
\text{getMthd}(X, C, \text{parent}(C), C, C)
\]

(3.8)

\[
\text{parent} : \text{Name} \times \text{ClassSet} \rightarrow \text{Name}
\]

\[
\text{flds} : \text{Name} \times \text{ClassSet} \rightarrow \text{NameList}
\]

\[
\text{getInheritsSet} : \text{Name} \times \text{ClassSet} \times \text{ClassSet} \rightarrow \text{ComputationItem}
\]

\[
\text{getMthd} : \text{Name} \times \text{Name} \times \text{ClassSet} \rightarrow \text{ComputationItem}
\]

Figure 3.7: K Definitions of Common Operators

Rule (3.3) acting as an accessor to retrieve the parent class for class \( C \) and Rule (3.4) retrieving the fields for class \( C \). To get the set of classes inherited by a given class, we can work our way back through parent classes until we reach the Object class, the root of the class hierarchy. In Rule (3.6), class name \( C \) is added to the set and \( C \) is replaced by \( C \)'s parent. In Rule (3.5), where the root of the inheritance tree has been reached, Object is added to the set and the set replaces getInheritsSet on the computation. Thus, the set is built up in an iterative fashion and then returned. The definition of getMthd is straightforward; since KOOL does not allow overloaded method names and uses single inheritance, it is sufficient to check in each class up to the root for a method with the same name, returning the first found. If no matching method is found, an exception (not shown here) is thrown.

3.4.2 Program Evaluation

To evaluate a program in KOOL, the program must be inserted into an initial state on which the rewrite process can be started. The state will then proceed through a number of transitions until it reaches a final state (assuming it terminates), which could represent either an error execution, such as one in which an exception is thrown but not caught, causing the program to crash, or a successful execution, yielding some final output and no further execution steps. This is modeled using an \textit{eval} function, shown in Figure 3.8. Note that the function takes the program and the program input, and then provides default values for all other state components. The semantics will process all class definitions in the program.
within the $cset$ and execute the program expression. Since there are no features yet in the language that can introduce nondeterminism, a given program will always yield the same final state, with the final result in $output$, if it terminates.

### 3.4.3 Object Creation

Since all values in KOOL are objects, object creation is one of the core sets of rules in the semantics. At a high level, several distinct steps need to be performed:

- Since each class that makes up the object’s type – the current class and all superclasses up to and including $Object$ – can contain declarations, and since any of these declarations could be used, depending on the method invoked and the current scope, a “layer” for each class that makes up the object needs to be allocated, containing the layer name and name/location mappings for all instance variables;

- the layers need to be combined into a single object such that lookups occur correctly; specifically, lookups should start at the correct layer, based on the static scoping rules for the language;

- the newly created object, with the various layers and information about its dynamic class, then needs to be returned.

The rules for object creation are shown in Figure 3.9, with a visual example provided in Figure 3.10. Rule (3.10) handles the `new` expression. `new` is provided a class name ($C$) and a possibly empty list of arguments ($El$) to be provided to the class constructor. The desired result is that a new object of class $C$ will be created and the class constructor for $C$, which must also be named $C$, will be invoked on the newly created object. The `createObj` operation indicates that we want to create a new object of class $C$; this is included in a list with the arguments $El$ on top of the computation to make sure these are evaluated as well. The `invokeAndReturnObj` is beneath these waiting for them to yield a list of values; `invokeAndReturnObj` will then cause a method $C$ to be invoked on the newly-created object with the values resulting from evaluating $El$ passed as the actual parameters. We want to ensure that the object being created is returned at the end of this process; how this is handled can be seen in Rule (3.11), where `invokeAndReturnObj` is just replaced with an invoke of the same method, a discard to remove the value returned by the method, and finally the
target object, effectively replacing the return value of the method with the target object. So, this will take the new object, send it the constructor message with the provided arguments, and return the object, which is what we need. More details about handling message sends are provided in Section 3.4.4.

The rules that actually create the object start with Rule (3.12). As we create each layer, we want to allocate space for any field names which become visible at this layer. By default, this adds the names to the environment. To ensure we don’t leak names out, or add names in inadvertently, we first want to save the environment so we can recover it when we are finished and also clear it, so we start with an empty environment and just include field names. This is done by putting the environment \( Env \) on the computation (when an environment is encountered at the top of the computation it is recovered) and setting the \( env \) state attribute to \( \cdot \). Also, the createObj computation item is changed to a mkObj computation item, which contains two elements: the current layer that is being built and the object that has been constructed so far. The object, also represented as a set, is initialized with the dynamic class, which matches the class name in the new statement, and a default environment for Object, which is empty since Object has no fields. We set the current layer being built to the dynamic class of the object, since we need to start with this layer and work up the inheritance tree towards Object.

Now, we construct the object in an iterative fashion. Rule (3.13) shows the base case of the recursive creation, which is when we reach class Object. Here, we just take the current object and return this as the result of mkObj. Rules (3.14) and (3.15) show how the environment layers are configured for classes other than Object. In Rule (3.14), for class \( C \), we want to allocate space for all fields in the class and store them in the environment layer assigned to this class in the object being created. To allocate space for the fields, the bind computation

\[
\text{new } C(E) \\
\begin{array}{l}
\text{createObj}(C), El \rightarrow \text{invokeAndReturnObj}(C) \\
\text{invoke}(C) \rightarrow \text{discard} \rightarrow O \\
\text{createObj}(C) \rightarrow \text{mkObj}(C, \text{myclass}(C)) \rightarrow Env \\
\text{mkObj}(Object, O) \\
\text{bind}(flds(C, CSet)) \rightarrow \text{layer} \\
\end{array}
\]  

\[
\text{new}(\_): \quad \text{Name} \times \text{ExpressionList} \rightarrow \text{Expression}
\]

Figure 3.9: KOOL Object Creation Rules
item is used. This item is defined to take a list of names, add the names to the environment, and allocate storage for each name. Since there are no values on top of the bind, each name will be assigned the initial value nil in the store. \( \text{flds} \) is used to retrieve the fields of class \( C \), as defined in class set \( CSet \). The \textit{layer} computation item then says that a new layer should be formed from the resulting environment.

The process of forming this layer is shown in Rule (3.15), where the current environment is added into the object environment \( (OE) \) inside the object as \( [C,Env] \), or the environment layer associated with class \( C \). The environment is then cleared out, and the process is continued with the parent class of \( C \). Eventually this will reach Rule (3.13), return the object, call the constructor, and yield a new, initialized object.

In summary, for each layer, we grab back the fields available in the class for that layer. We then allocate space for them, and initialize them to nil. Finally, we save this environment, which just contains the field names and memory locations for this layer, into the object environment \( (oenv) \), tagging them with this layer’s name, then clear out the environment and continue by adding a layer for the parent.

Rules (3.3) and (3.4) show the process of getting the parent class and the fields for a given class and class set, respectively. In Rule (3.3), the class name is used to match against the parent class name in the set representing the class, while in Rule (3.4) the class name instead matches against the list of names representing the fields of the class.
An example object creation can be seen in Figure 3.10. The class, ColorPoint, contains two fields, c and p. It extends class Point, which contains three fields, x, y, and p. This class extends Object by default, which has no fields. As can be seen in the Figure, the computation item createObj(ColorPoint) will lead to the computation item mkObj with the initial class and an initial version of the object. Each step will then either bind fields from the class or add those fields as a new layer in the object environment. Note that there are two copies of field p, one at location L3 and one at location L5. The copy chosen will depend on the method being executed – a method from class Point will use the copy of p at L5, while a method from class ColorPoint will use the copy of p at L3. Once the creation reaches Object, the new object has been created and is returned.

The next step, sending the ColorPoint message with the constructor arguments, is not shown.

### 3.4.4 Message Sends

Message sends use dynamic dispatch by default in KOOL. Because of this, lookups for the correct method to invoke should always start with the dynamic class of the object, working back up the inheritance tree towards the Object class. There are two exceptions to this rule. First, with super calls, the correct instance of the method to call should be found by starting the search in the parent class of the current class in the execution context (in other words, the parent class of the class which contains the currently executing method). Second, with constructor calls, the lookup order is the same, but the method name will change, since constructor method names match the class name in which they are defined. The first exception is part of the semantics for super, which are separate; the second is part of the core send semantics, but is not shown here. The rules for message sends are shown in Figure 3.11.

The first rule, Rule (3.16), is used to start processing the message send. The message target, E, and the message parameters, El, are evaluated, with the name of the message, X, saved in the invoke computation item. In Rule (3.17), given the result of the evaluation of E and El, the current stream of execution from the computation (K), the control state (Ctrl), and the current environment (Env), object (O'), and class (C') are pushed onto the method stack (with the environment on top of the remaining computation, so it will be recovered when this computation is run), ensuring that the current execution context can be quickly restored when the method exits. The computation is changed to put the value list (Vl) that resulted from evaluating the message parameters on top of the getMthd computation item, which is on top of a different invoke computation item that takes no parameters. This indicates that we want to find the method to invoke, based on the method name, class name, and class set, and then invoke it with actual arguments Vl. The environment is cleared to ensure names in the current environment aren’t introduced into the environment of the
\[
E.X(EI) \\
(E, EI) \mapsto \text{invoke}(X)
\]

Rule (3.16) shows the result of finding the method. A pair of the class name in which the method was found and the method itself are on top of the `invoke` computation item. This will be replaced with a bind of the method parameters and declarations \((Xl,Xl')\), followed by the method body \((K')\). The values in \(Vl\) will then be bound to the names in \(Xl\), with the declarations \(Xl'\) bound to \text{nil}, giving us the proper starting state for executing the method body (by default declarations are assigned a value of \text{nil} until they are assigned into). The class context is changed to the class, \(C\), in which the method was found.

Rule (3.19) shows the result of reaching the end of a method. All methods are automatically ended with a “\text{return nil;}” statement when they are preprocessed, so even method bodies without an explicit \text{return} will eventually encounter one. When \text{return} is encountered, the \text{return} computation item and the rest of the computation following \text{return} are discarded, replaced by the computation on the method stack. The rest of the control state, the current object, and the current class are also reset to the values from the method stack. This will set the execution context back to what it was at the time the message was sent – back to the context of the invoking object. The value on top of the computation is left untouched, however, since this will be returned as the result of the message send.

Figure 3.11: Message send rules
3.4.5 Exceptions

KOOL includes a basic exception mechanism similar to that in many other OO languages, such as Java or C++. Code can be executed in a try block, which has an associated catch block. When an exception occurs, control is transferred to the first catch block encountered as the execution stack is unwound. The exception, represented in KOOL as an object, is bound to a variable associated with the catch, with different classes of exceptions used for different exception conditions (nil reference, message not supported, etc.). Along with system-defined exceptions, custom exception classes can be created, and both can be thrown using a throw statement. The semantics for exceptions can be seen in Figure 3.12. One important point is that exceptions are not just added by the programmer – they are used in the language semantics as well. For instance, although not shown in the rules for message sends, several possible exceptions can be raised, including an exception generated when a nil variable is used as a message target and an exception thrown when a target object does not support a message (the name and arity must match those in the call). An example where an exception is thrown by the semantics rules can be seen in Figure 3.17 in Section 3.5, where an exception is thrown on a lock release when the lock was not already held.

Rule (3.20) shows the semantics for a try-catch statement. The current control context (Ctrl), environment (Env), object (O), and class (C), along with an exception computation, are all put onto the exception stack. The exception computation is made up of a binding to the name X from the catch clause, the statement S' associated with the catch clause, the current environment Env (so we recover the current environment and remove the binding of the
caught exception to \( X \), and the current computation, \( K \). Finally, the try-catch block is replaced with the statement \((S)\) from the try clause and the popEStack computation item. So, for a try-catch block, we will execute the statement in the try clause. If this finishes, we will pop the exception stack and continue running. If an exception is thrown, we will instead want to execute the catch clause, binding the exception to the name in the clause, running the body of the catch, and then continuing with the remainder of the computation after the end of the try-catch statement.

Rule (3.21) handles the no exceptions case, where the pop marker is found during normal execution. In this case, the top of the exception stack is popped, but no other changes occur. When an exception is thrown, Rule (3.22) is used. In this case, the current context information is replaced with the information that was saved on the exception stack, and the exception stack is popped, essentially "unrolling" the execution stack in one shot. The value \( V \) that represents the exception is left on top, which will cause it to be bound correctly to the catch variable and made available to the catch statement (in Rule (3.20) the top of the stored computation was a bind, so the value will be bound to the name from the catch clause). Since the rest of the computation after the end of the try-catch statement was saved as part of the exception computation, the computation will continue correctly after the end of the exception handler.

### 3.4.6 Runtime Type Inspection

KOOL allows the dynamic type of an expression to be checked at runtime using a typecase construct. This construct contains a sequence of cases, each with a class name and a statement. If the class name in the case matches either the dynamic class type of the expression or a superclass of the dynamic class type, the statement is executed. Cases are evaluated from top to bottom, with an optional else case that always matches. The rules for runtime type inspection are shown in Figure 3.13.

Since the parsing step can convert the else case to a case matching Object,
we assume in the semantics that there is no longer a designated else case. When a typecase is encountered, Rule (3.23) shows that this is replaced with an evaluation of the expression $E$, on top of the getInheritsSet computation item, followed by the Cases that will be checked. When the expression $E$ is evaluated to an object value, Rule (3.24) shows the start of building the set of class names that will be used in the check against the cases. The getInheritsSet computation item is changed to another item with the same name but three parameters, a class name, a set of class names and a set of classes, with the first two parameters set to the dynamic class of the expression result, $C$. The inherits set is built according to the rules in Figure 3.7.

With the set of classes for the expression calculated, the remaining three rules, Rules (3.25), (3.26) and (3.27), process the cases. In the first, a matching case in the set of cases (the rest of the set is $\_\_\$) is found, so the class name set $(C, \_\_\$) and the remainder of the cases list are both discarded, replaced by the statement $S$ from the matched case. In the second, the case does not match, but there are cases left in the list, so the current case is removed, allowing the next to be tried. In the third, there are no cases left in the list, so both the cases list and the class name set are discarded, allowing control to fall through to whatever was after the case statement. This provides for the intended semantics – the statement of the first matching case (if any) will execute, then control will pick up with the next statement after the end of the typecase.

3.4.7 Primitives

Since all operations are modeled as message sends, there isn’t a native way in the language to, for instance, add two numbers, or output a string. Yet, at some point, $5 + 3$ actually has to yield $8$. This is done using primitives, a concept similar to that used in Smalltalk. Each class which is used to represent a primitive value, such as Integer, is declared as a primclass, which adds a field that stores the primitive value. This field can be accessed by the primitive operations to either take out the existing primitive value or put a new one in. For instance, for $5 + 3$, primitive operations would take out the value $5$ and the value $3$, add them using the system version of integer addition, create a new Integer object, and put the primitive value $8$ into the new object’s primitive value field. All “system” operations, including input and output, are handled using primitives, providing the programmer with an object-level view of the primitive operations.

3.5 Adding Concurrency

The dynamic semantics from Section 3.4 does not support any concurrent operations – as defined, KOOL is a sequential language, with a single thread of execution. In this section language prototyping is illustrated by extending
KOOL with native support for concurrency. There are many different options for how concurrency can be supported. The model chosen here is simple on purpose, but still provides enough flexibility to create programs with multiple concurrent threads and thread synchronization operations.

To support concurrency, a new statement, spawn, is added to the syntax to allow the creation of new threads. Threads will be able to acquire and release locks on specific objects (similarly to the basic locking functionality in the Java language) using acquire and release statements. Finally, the semantics should correctly model the accesses to shared memory locations, which should compete – if two threads both assign a value to a shared variable, the resulting value should be nondeterministic, based on the actual execution order of the threads.

With multiple threads, and thus multiple concurrent streams of execution, some of the state components will need to be duplicated. This includes any components which provide context to the current thread of execution: the current object, the current class, the entire control, and the environment. This allows each thread to have enough local information to execute without interfering with the execution of other threads. For instance, threads should not share the current class, since a message send in one thread would potentially interfere with a message send in the other if they did. However, some information, such as the
set of classes and the store, will be global to all threads. Figure 3.15 shows the configuration from Figure 3.16 extended to enable concurrency.

The additional syntax and new rules for the dynamic semantics for concurrency in KOOL are shown in Figures 3.14, 3.16, and 3.17. It is important to note that, even with fairly significant changes to the configuration and to the language functionality, most of the rules are new – the only changed rule is Rule (3.28), used for program evaluation, which does change to take account of the new state infrastructure. This is the concurrent version of Rule (3.9). Rule (3.28) makes use of the newThrd computation item to create a new execution thread and set up the starting state appropriately.

The spawn statement creates a new thread based on a provided expression. The expression is evaluated in the new thread, meaning any exceptions thrown by the expression when it is evaluated will be handled in the new, not the spawn’ing, thread. This is a design decision, and has been made to simplify the semantics; an earlier version of this rule assumed that only method calls could be spawned, and evaluated all method arguments in the current thread, providing different exception behavior. Rule (3.29) shows the semantics of spawn. Here, the expression E in the spawn statement is given to the newThrd item, along with the current environment (Env), the current object (O), and the current class (C). spawn returns no value, so it is just removed from the computation. Rule (3.30) shows how the new thread is actually created. The passed values for expression, environment, object, and class are plugged into the proper state components nested within the new thread. This will start the new thread for expression E running in the proper environment. When the thread finishes, it needs to be removed, with any locks it holds being removed from the global busy lock set. This is illustrated in Rule (3.31).

Along with the ability to create new threads, we also need to be able to
acquire and release locks. This is done using the acquire and release statements. The semantics for acquire is shown in Rules (3.32) and (3.33). In Rule (3.32), a lock is acquired on an object $V$ that the current thread already holds a lock on. This just increments the lock count on this object from $N$ to the successor of $N$. In Rule (3.33), a lock is acquired on a value $V$ that no thread, including the current thread, has a lock on. This adds the value $V$ and a lock count of 1 to the thread’s holds set, while also adding $V$ to the current global lock set $LS$. The lock count in holds is necessary to ensure that lock acquires and releases are balanced – a thread can acquire a lock multiple times, with a recursive method call for instance, and we need to ensure that a lock is not inadvertently released too soon.

The semantics for release is shown in Rules (3.34), (3.35), and (3.36). In Rule (3.34), a lock on value $V$ with lock count 1 is released. This removes the lock from both the local holds set and the global busy set. Rule (3.35) shows what happens when a lock on value $V$ with lock count greater than 1 is released – here, the count simply goes from $s(N)$ (the successor of $N$) to $N$. Finally, if there is an attempt to release a lock that the thread does not hold, an exception should be thrown. This is shown in Rule (3.36), where an attempt to release a lock on $V$ not held by the thread results in a LockNotHeldEx exception being thrown.

A sample concurrent program, the thread game, is shown in Figure 3.18. In this program a new class, ThreadGame, is defined. The constructor for this class sets field $x$ to the value 1. The Add method then includes an infinite loop that, during each execution of the loop body, issues a single statement, adding $x$ to $x$ and assigning the result back to $x$.

If this program were not concurrent, this would just double the value of $x$ each time through the loop. However, the Run method spawns two threads, each of which will execute the Add method. Because there is no synchronization used to prevent data races, the two threads can easily interfere with one another. In

$$
\langle k \rangle \text{acquire } V \ldots \langle k \rangle \langle \text{holds} \rangle \ldots \langle V, N \rangle \ldots \langle /\text{holds} \rangle \hspace{1cm} (3.32)
$$

$$
\langle k \rangle \text{acquire } V \ldots \langle /k \rangle \langle \text{holds} \rangle \ldots \langle /\text{holds} \rangle \langle /\text{busy} \rangle \ldots \langle LS \rangle \ldots \langle /\text{busy} \rangle \text{ if } V \notin LS \hspace{1cm} (3.33)
$$

$$
\langle k \rangle \text{release } V \ldots \langle k \rangle \langle \text{holds} \rangle \ldots \langle V, 1 \rangle \ldots \langle /\text{holds} \rangle \langle /\text{busy} \rangle \ldots \langle V \rangle \langle /\text{busy} \rangle \hspace{1cm} (3.34)
$$

$$
\langle k \rangle \text{release } V \ldots \langle /k \rangle \langle \text{holds} \rangle \ldots \langle V, s(N) \rangle \ldots \langle /\text{holds} \rangle \hspace{1cm} (3.35)
$$

$$
\langle k \rangle \text{release } V \ldots \langle /k \rangle \langle \text{holds} \rangle \text{LTS}(\langle /\text{holds} \rangle \text{ otherwise}) \hspace{1cm} (3.36)
$$

spawn : Expression $\rightarrow$ Statement
acquire : Expression $\rightarrow$ Statement [strict]
release : Expression $\rightarrow$ Statement [strict]

Figure 3.17: Concurrent KOOL Rules, Part 2
class ThreadGame is
    var x;

    method ThreadGame is
        x <- 1;
    end

    method Add is
        while true do x <- x + x; od
    end

    method Run is
        spawn(self.Add); spawn(self.Add);
    end
end

(new ThreadGame).Run

./runkool examples/ThreadGame.kool -t 10
... term omitted ...
Solution 1 (state 2294)
states: 3381 rewrites: 310427 in 14388ms cpu
SL:[StringList] --> "10"

Figure 3.18: The Thread Game in KOOL

fact, it has been proved that the variable x can take the value of any natural
number greater than 0 \[130]. Figure 3.18 also shows a sample run which checks
to see if the value 10 is reachable; this is found after generating 3381 reachable
states.

3.6 Other Extensions

Because of its use in our language research and in teaching, the KOOL system has
been designed to be extensible. This section illustrates two additional extensions
to KOOL: synchronized methods and label statements.

\[
\text{Method} \quad M ::= \quad \text{synchronized method } X \text{ is } D^+ S \text{ end} \mid \text{synchronized method } X (\{X',\}^+) \text{ is } D^+ S \text{ end}
\]

\[
\text{Statement} \quad S ::= X:
\]

Figure 3.19: KOOL Syntax, Additional Extensions
Figure 3.20: Synchronized Methods

3.6.1 Synchronized Methods

As shown in Section 3.5, the KOOL language has a fairly simple model of concurrency based on threads and object locks. Synchronized methods, similar to those in Java, would provide a higher-level abstraction over these locking primitives. In Java, methods tagged with the synchronized keyword implicitly lock the object that is the target of the method call, allowing only one thread to be active in all synchronized methods on a given object at a time. We will assume the same semantics for KOOL.

The syntax changes to add synchronized methods are minor: the keyword needs to be added to the method syntax, which then also needs to be reflected in the Maude-level syntax for KOOL. The extended concrete syntax for methods is shown in Figure 3.19. The abstract syntax is similar, with a new method operator, syncmethod:

\[
\text{op syncmethod ('_')is__end : Name Names Decls Stmt \to Method .}
\]

The changes to the semantics are obviously more involved. In KOOL, as in Java, synchronized methods should work as follows:

- a call to a synchronized method should implicitly acquire a lock on the message target before the method body is executed;
- a return from a synchronized method should release this lock;
- additional calls to synchronized methods on the same target should be allowed in the same thread;
- exceptional returns from methods should release any locks implicitly acquired when the method was called.

A quick survey of these requirements shows that adding synchronized methods will change more than the message send semantics – the semantics for exceptions will need to change as well, to account for the last requirement.

To handle the first requirement, a new lock can be acquired on the self object at the start of any synchronized method simply by adding a lock acquisition to the start of the method body, which can be done when the method definition...
is processed. Item (3.37) shows the abstract syntax of a non-synchronized method, with item (3.38) showing the constructed computation for executing the method body. Here, the body statement $S$ is executed, and then $nil$ is returned, ensuring that the method always returns some value. Item (3.39) then shows the abstract syntax for a synchronized method; note the only difference is the use of `syncmethod` instead of `method`. The computation built is different, though, as shown in item (3.40): first `self` is evaluated, and is then handled by the `acquireMthd` computation item. The definition of this item is shown in Rule (3.41). `self` will evaluate to some value $V$, which is then used by `acquire` to acquire a lock on the current object.

Unlike lock acquisition, lock release cannot be handled as simply, say by just adding a `release` to the end of the method computation. This is because there may be multiple exits from a method, including `return` statements and exceptional returns. Because of this, locks acquired on method entry will need to be tracked so they can be properly released on exit. This can be accomplished by recording the lock information in the method and exception stacks (`mstack` and `estack` in Figure 3.6) when the lock is acquired, since these stacks are accessed in the method return and exception handling semantics. With this in place, the second and fourth requirements can be handled by using this recorded information to release the locks on method return or when an exception is thrown. Looking again at Rule (3.41), the `addELock` and `addMLock` computations do exactly this, recording the lock information in the appropriate stacks for use later (the later definitions are not shown here).

Finally, the third point is naturally satisfied by the existing concurrency semantics, which allow multiple locks on the same object (here, `self`) by the same thread.

Overall, adding synchronized methods to the KOOL semantics requires:

- 2 modified operators (to add locks to the two stack definitions),
- 4 modified equations (two for method return, two for exception handling, using the locks added using `addELock` and `addMLock`),
- 4 new operators (to record locks in the stacks, to release all recorded locks),
- 6 new equations (to record locks in the stacks, to release all recorded locks).

The Maude search functionality can be quite helpful to gain confidence that new features are working as expected. For instance, an example of synchronized methods in KOOL is shown in Figure 3.21. Here, class `WriteNum` contains two synchronized methods. When the `write` method is called, the starting value of the number stored in member variable `num` is written to the console, some simple arithmetic operations are performed on it, and then the final value is written. The `set` method assigns a new value to `num`. Since both methods are marked `synchronized`, it should be the case that, for any given object, once one thread
class WriteNum is
  var num;
  
  method WriteNum(n) is
    num <- n;
  end

  synchronized method set(n) is
    num <- n;
  end

  synchronized method write is
    console << "Start:" << num;
    self.set(num + 10);
    self.set(num - 8);
    console << "End:" << num;
  end
end

class Driver is
  method run is
    var w1;
    w1 <- new WriteNum(10);
    spawn (w1.write);
    w1.set(20);
    spawn (w1.write);
  end
end

(new Driver).run

Figure 3.21: Synchronized Methods in KOOL

is executing either method, another thread that tries to execute either will wait. To test this, the Driver class creates a new object of class WriteNum, spawns one call to write, creating a new thread, modifies the value stored in the object using set, and then creates a second thread, also calling write.

Using search to determine possible program outputs reveals that there are only two possible solutions, with the call to set either occurring before the first spawned thread runs (with output "Start:", "20", "End:", "22", "Start:", "22", "End:", "24"), or after it completes (printing "Start:", "10", "End:", "12", "Start:", "20", "End:", "22"); this is shown in Figure 3.22. By contrast, with the synchronized keywords removed, there are 470 solutions, corresponding

> runkool -s --final Sync5.kool

Solution 1 (state 96)
states: 98 rewrites: 10390 in 612ms cpu (612ms real)
(16976 rewrites/second)
SL:[StringList] --> "Start:","20","End:","22","Start:","22","End:","24"

Solution 2 (state 97)
states: 98 rewrites: 10390 in 612ms cpu (612ms real)
(16976 rewrites/second)
SL:[StringList] --> "Start:","10","End:","12","Start:","20","End:","22"

No more solutions.

Figure 3.22: Search Results, With Synchronization
> runkool -s --final Sync6.kool

Solution 1 (state 80383)
states: 80853 rewrites: 10112671 in 671633ms cpu (674345ms real)
         (15056 rewrites/second)
SL:[StringList] --> "Start:","20","End:","22","Start:","22","End:","24"
...

Solution 470 (state 80852)
states: 80853 rewrites: 10112671 in 671645ms cpu (674360ms real)
         (15056 rewrites/second)
SL:[StringList] --> "Start:","10","End:","Start:","20","End:","22","12"

No more solutions.

Figure 3.23: Search Results, Without Synchronization

to all possible orderings of output based on various interleavings of the main
thread with the two spawned threads. This is shown in Figure 3.23.

3.6.2 Labels
To make it easier to write LTL formulae that refer to positions within a program,
label statements have been added to KOOL. A label statement is just an identifier
followed by a colon, such as X:. When a label is encountered, a cell, at the
thread level, that holds the current label is changed, allowing the change to be
captured in a proposition defined as part of KOOL’s model checking support.
This is shown in Figure 3.24, Rule (3.42).

In Maude, this is supported using the following model checking proposition:

\[
\text{op labeled : } \text{Nat Name} \rightarrow \text{Prop} \\
\text{eq t(tid(N) lbl(X) TS) S |= labeled(N,X) = true.}
\]

3.7 KOOL Implementation

KOOL programs are generally run using the runkool script, since running a
KOOL program involves several steps and tools, and since programs can be run
in different modes (execution, search, and model checking, each with various
options). First, the KOOL prelude, a shared set of classes for use in user
programs, is added to the input program. This program is then parsed using

\[
\langle k \rangle \text{label(X) ...} \langle /k \rangle \langle lbl \rangle ... \langle lbl \rangle \\
X
\]

Figure 3.24: Label Statements
the SDF parser [180], which takes the program text and a syntax definition file as input. The parser produces a file in ATerm format, an SDF format used to represent the abstract syntax tree. A pretty printer then converts this into Maude, using prefix versions of the operators to prevent parsing difficulties. Finally, Maude is invoked by runkool with the language semantics and the Maude-format program, generating the result based on the execution mode. A graphical view of this process is presented in Figure 3.25.
Chapter 4

A Prototype of Beta

KOOL was designed using prototyping techniques, and was also designed as a platform for experimenting with language features. To show that the techniques used for experimenting with the KOOL definition can also be used for programming languages not designed with these techniques in mind, this chapter presents parts of a definition of the Beta language [113]. At the time this material was being written the Beta definition was being modified to take advantage of techniques developed over the last few years in K and rewriting logic semantics. The version presented here is based on an earlier version of the language definition [75], although the semantic rules show here are presented using the newer K format. Information about the version currently being developed can be found online at the FSL Beta language website [74].

4.1 The Beta Language

Beta is an object-oriented language developed as the successor to the Simula 67 language [147]. It is probably best known for two features that set it apart from more commonly used object-oriented languages, such as C++ and Java:

- Beta has collapsed the concepts of class and method into a single concept, the pattern. A pattern consists of local variable declarations, input parameters (the enter block), output parameters (the exit block), and actions (the do block). Instances of patterns are created to represent objects, execute as methods, etc.

- To invoke inherited but overridden functionality, most object-oriented languages provide a super-call mechanism. Beta does the opposite: method dispatch always starts at the top of the inheritance hierarchy, and then heads down the hierarchy towards the actual pattern using inner calls. This provides a method to enforce behavioral subtyping [110], since a parent pattern can either choose not to invoke a child pattern, or can at least ensure that necessary invariants still hold after the child executes.

Both of these features, along with a number of interesting, yet more common, language features, such as deterministic alternation (i.e., coroutines) and concurrency, make Beta a challenging language to correctly define.
4.2 Beta Semantics

The dynamic semantics of Beta are specified using K, translated into Maude to provide an environment for program evaluation and analysis. In this section several of the more interesting features of the definition are presented. The remainder of the definition is available on the FSL Beta website [74].

4.2.1 Method Dispatch Semantics

Similar to C++, Beta provides both static and dynamic dispatch for invocations of patterns used like methods. Standard pattern declarations make use of a colon to separate the name of the pattern from the implementation:

\[
\text{DisplayReservation:}\nonumber
\begin{align*}
\text{do Date.Display; Customer.Display; INNER} \\
\end{align*}
\]

Virtual patterns use similar notation, with :< indicating that a pattern is being made virtual in this pattern and all inheriting patterns:

\[
\text{Display:< DisplayReservation}
\]

In child patterns that provide a new version of the virtual pattern, the notation ::< is used instead. This first example provides a new version of the Display pattern, in this case overridden to properly handle displaying train reservations:

\[
\text{DisplayTrainReservation: DisplayReservation}\nonumber
\begin{align*}
\text{do ReservedTrain.Display;} \\
\text{ReservedCarriage.Display;} \\
\text{ReservedSeat.Display;} \\
\text{INNER} \\
\end{align*}
\]

\[
\text{Display::< DisplayTrainReservation}
\]

The second example provides a different extension of the basic reservation display functionality, this time for flights:

\[
\text{DisplayFlightReservation: DisplayReservation}\nonumber
\begin{align*}
\text{do ReservedFlight.Display;} \\
\text{ReservedSeat.Display;}
\end{align*}
\]

---

[1] The sample code shown in this section is from the book “Object-Oriented Programming in the BETA Programming Language”, by Madsen, Møller-Pedersen, and Nygaard [113]
Rules to determine which pattern is actually invoked are then based on the type of the variable used for invocation. When reference variables are declared, they are declared with an initial type, say $T$. They can then hold an instance of a pattern of type $T$ or of a pattern which inherits from type $T$, say $T'$. Only patterns declared in $T$ or a parent of $T$ can be invoked through the reference. If a pattern, say $m$, is visible in the declaration of $T$ and is statically dispatched we want to invoke the closest instance of $m$ to $T$ declared between $T$ and the root of the inheritance hierarchy, $\texttt{Object}$. If $m$ is marked as dynamic, we will perform a similar search, but instead we will start at $T'$, the dynamic type of the object instance.

Note that, in these rules and rules shown later in this section, pattern types $T$ are assumed to be identifiers which point to pattern definitions visible in the current environment. Inserted items, which are given by writing the code for the pattern inline, can be handled by assigning them a unique name, allowing them to be treated the same as named patterns. It is assumed in the rules given below that this has already been done.

Figure 4.1 shows several rules used to look up the correct version of an invoked pattern, with support for dynamic dispatch. Several operators are used in the rules, either to act as part of the computation or to track information used in the semantics. $\texttt{href}$ represents a reference variable (it holds a reference), and holds both the pattern type $T$ given in the variable declaration and the currently assigned object reference $\texttt{oref}(L)$. The $\texttt{lookup}$ item holds the name being looked up in the pattern, which in the case of a pattern invocation would be the name of the pattern being invoked (in Java this would be the method name). $\texttt{plookup}(T)$ retrieves a pattern definition from the environment (Beta scoping rules allow multiple patterns to have the same name, with standard static name shadowing indicating which names are visible). $\texttt{block}$ is just used internally, to keep adjoining values in the computation from being collapsed into:

\[
\begin{align*}
\langle k \rangle &\quad \text{href}(T; \texttt{oref}(L)) \curvearrowright \texttt{lookup}(X) \\
&\quad \text{plookup}(T) \curvearrowright \text{block}((\text{href}(T; \texttt{oref}(L)))) \curvearrowright \text{lookup}(X) \\
\langle k \rangle &\quad \text{pt}(\text{pname}(Xc) P) \curvearrowright \text{block}((\text{href}(T; \texttt{oref}(L)))) \curvearrowright \text{lookup}(X) \\
&\quad \text{deref}(\texttt{oref}(L)) \curvearrowright \text{dispatch}(T; \texttt{oref}(L), \text{pt}(\text{pname}(Xc) P')) \curvearrowright \text{lookup}(X) \\
\langle k \rangle &\quad \text{pt}(\text{virtual}(\texttt{true}) P) \curvearrowright \text{dispatch}(T; \texttt{oref}(L), \text{pt}(P')) \curvearrowright \text{lookup}(X) \quad \langle k \rangle \\
&\quad \text{oref}(L) \curvearrowright \text{lookup}(X) \\
\langle k \rangle &\quad \text{pt}(\text{virtual}(\texttt{false}) P) \curvearrowright \text{dispatch}(T; \texttt{oref}(L), \text{pt}(P')) \curvearrowright \text{lookup}(X) \quad \langle k \rangle \\
&\quad \text{pt}(\text{virtual}(\texttt{false}) P')
\end{align*}
\]

Figure 4.1: Beta Semantics: Dynamic Pattern Lookup
a value list. The `deref` item retrieves the actual object stored at the location pointed to by `oref(L)`, and `dispatch` keeps track of the information we need for pattern dispatch: the type `T`, the object reference `oref(L)` we are dispatching to, and the pattern `pt(P')` that defines the type `T`.

With all this in mind, the rules in Figure 4.1 can now be understood. In Rule (4.1), when we are looking up a name (say `m`) via a reference (say with static type `T`), we first get back the pattern that defines type `T`. In Rule (4.2), with the pattern for `T` available, we use `deref` to retrieve the referenced object and store the information we will need to perform the pattern dispatch later, once the actual object has been retrieved. A rule not shown here uses the retrieved object to find the first definition of `m` at the level of `T` or above (towards `Object`).

Rules (4.3) and (4.4) then trigger the correct dispatch. In Rule (4.3), if the definition of `m` states that it is virtual, we will again look up `m`, but this time we will use the dynamic type of the referenced object, not the static (declared) type of the reference. In Rule (4.4), where virtual is false, we have already found the pattern that will be instantiated by this call, so we can just return that. In both cases, the expectation is that a pattern will be returned, which will be instantiated to invoke the pattern code.

### 4.2.2 Pattern Membership Semantics

In Beta, variables can be queried for the pattern they instantiate. This is called *pattern membership* and can be used to test (say, in a conditional with multiple branches) whether an object is an instance of one of several patterns:

```plaintext
(if R##
    // TrainReservation## then NTR+1->NTR
    // FlightReservation## then NFR+1->NFR
    if)
```

Here, the `##` operator is used with variable `R` to determine its pattern; the same operator is then used inside the conditional (the Beta conditional, in its general form, uses a number of cases, each preceded by `//`) in two branches, one branch testing to see if `R` is of pattern type `TrainReservation`, the other testing to see if `R` is of pattern type `FlightReservation`.

Figure 4.2 shows rules used to retrieve the pattern. Rule (4.5) provides the initial rule used by the `##` operator: given `E ##`, `E` is evaluated, and the pattern used to define `E` is then retrieved. Rules (4.6) and (4.7) handle the case where `E` is an object reference. In Rule (4.6), when an object reference is returned, the reference is dereferenced to retrieve the actual object. In Rule (4.7), once this object is retrieved, the object’s `myPattern` attribute is used to get back the actual pattern used to define the object. The use of `plookup` will retrieve the pattern itself. In Rule (4.8), the lookup resulted directly in a pattern (this would happen in cases such as `FlightReservation ##`), in which case that pattern is returned.
4.2.3 Code as Values Semantics

In Beta it is not only possible to determine the pattern of an expression, it is also possible to treat patterns as values which can be assigned to pattern variables and then instantiated like any other pattern. To do this, we can leverage other parts of the language definition, including definitions for assignment and pattern membership.

Rule (4.9) provides the base semantics for creating a pattern variable as an extension of other rules for creating declarations (not shown), which also use the createDec computation. Given a pattern type T, a new variable using operator pvar is created. This is similar to href, holding both the declared type of the pattern variable, which constrains what can be held, and the actual pattern, initially set to nothing.

Rule (4.10) extends the pattern membership test logic, discussed earlier, with support for pattern variables. Rule (4.11) then provides an extension to the logic used to look up a pattern for invocation, allowing pattern variables to be used in place of explicit pattern names. Finally, Rules (4.12) and (4.13) provide logic to handle assignments to pattern variables. Rule (4.12) triggers evaluation of both sides of the assignment (the assignment target is to the right of the arrow), indicating that this should be an assignment to a pattern variable. Rule (4.13) then specifies the logic used to assign the pattern: given an existing pattern variable pvar, and given that E evaluated to pattern P, pattern P is saved inside
the pvar, which is then assigned back to X.

4.2.4 Deterministic Alternation Semantics

The Beta deterministic alternation functionality allows Beta to support coroutines. Instead of using multiple threads, with objects running concurrently, different objects run inside the same thread, with each object running until it either finishes its do block or issues a suspend call.

The most interesting part of the alternation semantics deal with suspending an object and restarting it later. The semantics to do this are shown in Rule (4.14) in Figure 4.4.

This rule is triggered by a suspend call; a similar equation handles the restart. When suspended, the current execution stack needs to be saved, and the prior stack recovered. Several pieces of state information help with this process. altloc contains the memory location of the variable that holds the alternating object, which will need to be updated to hold the altered state. altstate contains the thread state that was current when the alternating execution of this object was started, which is the state that needs to be recovered to correctly handle the suspend. Note that it contains similar information to the current matched context, including its own version of the altstate, allowing nested alternations to be properly represented.

Now, using these parts of the configuration, the rule takes the current pieces of state that need to be saved at suspend, including the current computation (minus the saveAndRecoverStack item), current environment, and other parts of the thread state; puts these into a frozenState item; and assigns them back to the location holding the alternating object that is currently executing. Next, the remainder of the current computation is switched for the saved computation stored in altstate (suspending also passes back the exit values for the pattern, hence the need for Vl on top of the computation). At the same time, the rule recovers the old environment, altloc, and altstate, along with other thread state information, ensuring that when the computation continues it is back in the correct state.
4.3 Beta Implementation

The version of the Beta semantics described in this chapter uses an encoding of the Beta syntax directly into Maude notation, using Maude sorts, operations, and operator precedences. This has the advantage of allowing Beta programs to be written directly as Maude terms in a Beta-like syntax. However, it is also limiting in two ways. First, to allow Maude to disambiguate language constructs that look similar, it is often necessary to either add a large number of parentheses to programs or modify the syntax, giving different language constructs distinct syntactical representations. Second, it is not possible to take actual Beta programs and run them directly in the semantics, since they first have to be converted to work with the Maude version of the syntax.

The version of the Beta semantics currently being developed instead uses the Beta metaprogramming capabilities to define a Beta lexer, parser, and pretty-printer. The pretty-printer generates terms using a Maude-defined abstract syntax of Beta, transforming the constructs into the prefix versions of the defined mixfix syntax. By moving complex parsing outside of Maude, it is possible to use syntax closer to the actual Beta language when writing rules, while eliminating the cumbersome process of defining precedences in Maude. It is also possible to process actual Beta programs directly, making it easier to use the definition. Outside of these parsing and pretty-printing capabilities, the remainder of the system in the version currently being developed, including all semantics and analysis tools, is written in Maude.

4.4 Extending Beta

One goal of the work on language prototyping is to provide an environment where changes to a language can be implemented quickly, allowing for experimentation with new language features. As an example, we decided to add super calls to the Beta language, taking our inspiration from similar work which added inner calls to a Java-like object system in MzScheme. Since Beta has both static and virtual pattern dispatch, we have started by just adding this to static calls, but we believe it would be fairly straightforward to also add this feature to dynamic calls. We also have restricted this feature to named patterns only – it doesn’t seem as useful to allow this with anonymous patterns, since we cannot use an anonymous pattern as a parent to other patterns.

To add super calls, we first added a new type of pattern, a Java pattern, which can be the target of a super call. This required adding two syntax operators: one for pattern declaration, to indicate that the pattern being defined is a Java-style pattern; and one new expression, super, used the same as in Java, to initiate a super-call. We then needed to add two operators to the

\[^{2}\text{It is always possible to use a mixfix operator in prefix form; one advantage of this is that the prefix form is easier for Maude to parse, while the mixfix form can still be used when writing equations and rules in the semantics.}\]
semantics as well: a boolean flag, stored as part of the information kept with patterns, to specify whether the pattern is a Java-style or Beta-style pattern; and a computation item, used as part of the pattern semantics to appropriately set this boolean flag on newly created patterns.

In the language semantics, we changed a total of 9 rules and added 9 more. Of the 9 that were changed:

- 5 were to initialize patterns to not be Java patterns by default, but to act as standard Beta patterns;
- two were to properly handle building the inner call list used for pattern invocations;
- one was to trigger pattern invocations to start at Object if the pattern invocation is not of a Java pattern, i.e. to use standard Beta semantics;
- and one was to flag the \texttt{Object} pattern as a standard (not Java) Beta pattern.

Of the 9 rules that were added:

- 2 were to enumerate fields in a pattern with the new Java pattern syntax;
- one was to ensure Java patterns are not added to the list for inner calls;
- one was to ensure that a pattern invocation of a Java pattern would start at that pattern instead of at Object;
- two were to handle super calls;
- and three were to handle setting the Java flag on Java patterns.

Overall, it took roughly 2 hours to add this feature into our prior definition of Beta. It has not yet been ported forward to the new definition, but plans are to do so once the language definition is complete. The files with the extended semantics, with examples with and without \texttt{super}, are available on the rewriting logic semantics Beta website \cite{74}.
Chapter 5

The K Module System

One important aspect of formalisms for defining the semantics of programming languages is modularity. Modularity is generally expressed as the ability to add new language features, or modify existing features, without having to modify unrelated semantic rules. For instance, when designing a simple expression language, one may want to use structural operational semantics (SOS)\(^1\) to define the semantics of addition:

\[
\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2}
\]  

\(\text{(EXP-PLUS-L)}\)

\[
\frac{e_2 \rightarrow e'_2}{n_1 + e_2 \rightarrow n_1 + e'_2}
\]  

\(\text{(EXP-PLUS-R)}\)

\[
n_1 + n_2 \rightarrow n, \text{where } n \text{ is the sum of } n_1 \text{ and } n_2 \quad \text{(EXP-PLUS)}
\]

Further extending the language, one may want to add variables. One way to do this is to define an environment, mapping names to values, with rules for binding values to names (not shown here) and to retrieve the value of a binding:

\[
\langle x, \rho \rangle \rightarrow \langle n, \rho \rangle, \text{where } n = \rho(x)
\]  

\(\text{(VAR-LOOKUP)}\)

With this change to the language, even though the rules for plus do not actually reference the environment they are still modified to include it as part of the configuration. As an example, rule \text{EXP-PLUS-L} becomes:

\[
\frac{\langle e_1, \rho \rangle \rightarrow \langle e'_1, \rho \rangle}{\langle e_1 + e_2, \rho \rangle \rightarrow \langle e'_1 + e_2, \rho \rangle}
\]  

\(\text{(EXP-PLUS-L)}\)

To accommodate other language features that require extensions to the configuration, such as stores (for updatable memory locations) or mappings to information about classes and methods (in an object-oriented language, especially one where this information can change as the program runs, such as Self or Python), similar changes are made to the same language features, even in cases where they are not used directly in the rules. Alternatively, similar changes

\(^1\)The same rules are used in Chapter 9\]
may need to be made to add addition expressions to a different language with a
different configuration, even if the different elements of the configuration are not
used in the rules for addition. All these changes are required because SOS is not
modular. Improved support for modularity eliminates the need to make these
changes, offering several advantages:

- Modular definitions of language features allow other parts of a language
to change more easily by allowing existing feature definitions to remain
unchanged in the face of unrelated modifications or additions;

- A modular definition of a language feature can be more easily reused
in the definition of a different language, even one with a much different
configuration;

- Modular definitions are easier to understand, since the rules given for a
language construct only need to include the information needed by the
rule, instead of including extraneous information used in other parts of the
language (such as the store in the rules for plus).

For these reasons, improving modularity of language definitions has been a
focus of research across multiple semantic formalisms. One example is modular
structural operational semantics (MSOS) \[135\,137\], which solves the problem
shown above by leveraging the labels on rule transitions, not normally used in
SOS definitions of programming languages, to encode configuration elements,
with the ability to elide unused parts of the configuration. This, and other
related work in modularity, is discussed in Chapter 9.

With a tool supported semantics, modularity can also be expressed as the
ability to package language features into discreet reusable units, which can then
be assembled when defining a language. The ability to create reusable language
feature modules requires having a modular semantics, since it should be possible
to plug the same feature into multiple language definitions, even in cases where
(unused) parts of the configuration are different. Additionally, it should be
possible to provide clean interfaces to language features and to different parts of
the configuration, something not required in monolithic definitions, or even in
modular definitions written on paper.

Context transformers, described in Section 2.5, are targeted towards the first
definition of modularity, allowing rules to remain unchanged as new features
are added and as the configuration used to represent the programming language
state is changed. The K module system, described here, is targeted towards the
second, providing a method to package up K definitions into modules which can
be stored in a shared repository and then reused when creating or modifying a
language definition. Section 5.1 provides an overview of the K module system,
introducing the basic constructs used when creating modules. Section 5.2 then
illustrates these concepts in the context of a number of short examples. To show
a more complete application, Section 5.3 provides an example of a complete,
albeit simple, kernel of the C language, which is then extended with a new feature requiring changes to the configuration. Current K tool support uses Maude; Section 5.4 provides details of the current translation from K modules into a format usable by the current K implementation. Section 5.5 then discusses the online semantics module repository, which includes client-side tools, built around web services technology, and an initial XML document format designed to allow the open exchange of semantics modules. Finally, Section 5.6 presents a discussion of some of the challenges and limitations of the material presented here and in the last two chapters.

5.1 K Modules

The K module system provides a general module format designed to allow the definition of all features needed in a K language definition: abstract language syntax, configuration items, semantic rules and equations, and the ultimate assembly of a language, including the layout of the language configuration.

5.1.1 Module Notation

The module syntax is similar to that used in Maude, but with some simplifications and some extensions to capture common language definition scenarios. The notation is described below, grouped into constructs used in the module header and in the module body.

Module Headers

A module header consists of the name of the module, a number of import, export, and requires directives (one may include 0 or more of each), and a “wrapper” around the module body items, starting with begin and ending with endmodule. An overview of this notation is provided in Figure 5.1.

Module Name: In the K module system, module names are specified with a full module path, a /-separated list of individual names. Examples of valid

```k
module MODULE/NAME is
    imports IMPORTS .
    exports EXPORTS .
    requires REQUIRES .
    version VERSION# .
    description "Arbitrary string." .
begin
    module body items
endmodule
```

Figure 5.1: The Module Header
names would be \texttt{INT}, \texttt{EXP/PLUS}, and \texttt{IMP/BEXP/AND}. This use of paths provides a way to logically group modules. For instance, all feature definitions representing expressions could be given under \texttt{EXP}. Module names are not case sensitive, although they are traditionally given using upper case alphanumeric characters. The names are normalized inside the module system, so \texttt{Exp}, \texttt{exp}, and \texttt{EXP} are all considered to be the same module.

As well as the main module path, a module name may include an optional \textit{tag}, indicating the type of module being defined. This tag is user-defined, although two system-defined tags are also provided, \texttt{SYNTAX} and \texttt{LANGUAGE}. These indicate that a module defines either the abstract syntax of a language feature or an entire language, respectively. Examples include \texttt{EXP/PLUS[SYNTAX]} and \texttt{KOOL[LANGUAGE]}.

\textbf{Imports:} An \texttt{imports} clause indicates that a module depends on the definitions provided in the imported module. \texttt{imports} is equivalent to the Maude \texttt{including}; at this time there is no equivalent to either \texttt{protecting} or \texttt{extending}. Following the keyword \texttt{imports}, one or more modules may be listed by module path, each with an optional clause indicating modifications upon import. A standard example of an \texttt{imports} clause is the following:

\begin{verbatim}
    imports IMP/CONFIG/ENVIRONMENT .
\end{verbatim}

Tags can also be provided on import paths:

\begin{verbatim}
    imports IMP/EXP/PLUS[DYNAMIC] .
\end{verbatim}

Imports also provide an optional renaming clause, indicating that names of sorts or operations are being changed. The syntax for this clause is borrowed directly from Maude, and has been extended to also allow additions of K attributes, such as \texttt{strict}, to be defined on an operator on import. This allows an operator to be initially defined without attributes such as strictness, since different semantics may want to handle strictness in different ways.

### These represent a comment

### This import would be for a standard dynamic semantics,
### since we want to evaluate the guard but not the branches.

\begin{verbatim}
    imports FUN/EXP/IF[SYNTAX]
    * ( op if_then_else : Exp Exp Exp -> Exp now strict(1) ) .
\end{verbatim}

### This import would be for a static semantics, where we need
### to check all branches.

\begin{verbatim}
    imports FUN/EXP/IF[SYNTAX]
    * ( op if_then_else : Exp Exp Exp -> Exp now strict ) .
\end{verbatim}
Exports: An exports clause indicates that a module exports certain sorts, operators or K cells:

```
exports sort Env, op _[_] : Env Id -> Loc, cell env(Env) .
```

If no exports clause is included in the module header, all sorts, operators, and cells are exported from the module by default. If an exports clause is provided, any sorts, operators, or cells not listed are considered private to just that module, providing a method to create (for instance) local sorts and operations that do not conflict with other sorts and operators in other modules which may share the same name.

Requires: Using imports, it is possible for a module to include defined features, including operations, sorts, and cells, given in another module. To defer the decision of which module to import for these features, they can instead be listed in a requires clause. requires indicates that the features are used in the current module, but that an actual definition will be provided later, inside another module. For a module with a requires clause to be used in a language definition, all its requirements must be met by one or more imported modules.

```
requires sort Env, op _[_] : Env Id -> Loc, cell env(Env) .
```

Version: The version number is optional, and is currently used only when interacting with the module repository. The version number is in decimal format, with up to three digits following the decimal point: numbers like 1, or 1.01 are valid version numbers, but A3 or 1.1.3 are not.

Description: Like the version number, the description is optional, and is currently only used with the module repository. The description can contain an arbitrary string.

Module Bodies

The module body provides definitions of the items used in the syntax, semantics, and configuration of a language. The various types of module body items that can be defined are described below.

Sorts, Syntax Sorts, and Sort Aliases: Sort definitions in K modules use the same syntax as sort definitions in Maude, with the keyword sort (or sorts) followed by one or more identifiers naming the sort or sorts being defined:

```
sort Nat .
sorts Value ValueList .
```
To enable proper handling of definitional attributes such as strictness, it is important that K modules make a distinction between sorts used to represent abstract syntax and sorts used to represent non-syntax terms. Sorts defined using \textit{sort} are the latter; the keyword \texttt{xsort}, along with the variant \texttt{xsorts}, define the former:

\begin{verbatim}
  xsort Exp .  
  xsorts Stmt StmtList . 
\end{verbatim}

The K module system allows \textit{sort expressions} using a number of built-in \textit{sort constructors}, including constructors for maps, lists, and sets. Sometimes it is useful to define that a sort expression represents a specific concept, such as a list of values or an environment. This makes definitions more readable and provides better documentation, inside the definition, of the intent of the semantics designer.

\begin{verbatim}
  sortalias Env = KMap{Name,Loc} .  
  sortalias NumList = KList{Num} . 
\end{verbatim}

\textbf{Subsorts:} Subsorts are defined in K modules using the same syntax as provided by Maude, with the \texttt{<} operator used to separate space-separated lists of one or more sorts.

\begin{verbatim}
  subsort A B C < D E < F G . 
\end{verbatim}

\textbf{Ops and Syntax Ops:} Operators are defined in K modules using syntax that matches the syntax used in Maude. Like with sorts, it is important for K to be able to distinguish between operators representing abstract syntax terms and operators representing other operations in the semantics. This distinction is made by using \texttt{xop} (with variant \texttt{xops}) for abstract syntax operators, and \texttt{op} (\texttt{ops}) for the rest.

\begin{verbatim}
  xop _+_ : Exp Exp -> Exp .  
  op _[ ] : Env Id -> Location . 
\end{verbatim}

Several extended attributes for operators are also supported by the K module system. This includes \texttt{strict}, \texttt{seqstrict}, \texttt{renameTo}, \texttt{dissolve}, and \texttt{aux}. Other attributes available in the K Maude syntax are represented by explicit construct in the module system. For instance, the \texttt{wrapping} attribute, used in definitions of cells, is instead handled with explicit cell declarations.

\textbf{Vars and Var Prefixes:} The syntax for declaring variables in K modules is similar to the syntax for declaring variables in Maude. The keyword \texttt{var} (or \texttt{vars}) is followed by a space-separated list of variable names, with the sort given after a colon.

76
var N : Nat.
vars X X' Y : Id.

In place of a sort, a sort alias can be used. Also, it is possible to include sort expressions using K's predefined sort constructors as the sort of a variable.

sortalias Env = KMap{Name,Loc}.
var Env : Env.
var Mem : KMap{Loc,Value}.

In cases where many variables of the same sort would need to be defined, variable prefixes can be used instead. Variable prefixes allow the prefix of a variable name to be defined as being associated with a specific sort; any variable with this prefix will then be assumed to be of this sort assuming it is not redefined.

varprefix Env : Env.
vars N Env' : Nat.
...
  eq Env1 Env2 = ...
  eq N + Env' = ...

For instance, given a varprefix stating that all variables that start with Env are of sort Env, in the code above Env1 and Env2 are both considered to be of sort Env. Since Env' is specifically declared to be of sort Nat, it is proper to use Env' as a natural number in an equation that references it, such as the final equation that adds N and Env'.

Rules and Equations: Rules and equations can be written using either a traditional Maude format or the K contextual format. Maude-style rules and equations, including conditional variants, look just like standard Maude rules and equations:

eq s(N) + M = s(N + M).

cseq check(S SL) = check(S) check(SL)
  if SL =/= empty.

r1 A B C => X B.

crl S SL || T TL => SL || TL
  if equiv(S,T).

2The main difference between this and the equivalent syntax in Maude for using parameterized sorts is that the module system automatically imports the correct module with a definition of the sort and, in the translation into Maude, automatically creates the needed views. This is discussed further below.
K contextual rules use the special keywords `keq` and `krl` to define K equations and K rules, respectively. K rules and equations can both use the contextual, XML-like syntax provided by K to match across multiple cells in a computation:

```plaintext
keq <k> [[X.F(X1) ==> X1 -> K]] ...</k>
<mthds>... mthd(F,K) ...</mthds>.

krl <k> [[ X := V ==> .K]] ...</k>
<env>... [X,L] ...</env>
<mem>... [L,[[_ ==> V]]] ...</mem>.
```

The format of the rules and equations follows the format supported by the current version of the K tools package. Instead of underlining a part of the term, as described in Chapter 2, semantic-like brackets ([[ and ]]) are used along with the ==> arrow to indicate the before and after subterms (the subterms above and below the line). The other syntax was introduced in Chapter 2 as part of the K syntax: \(\ldots\) represents the unit; \(\_\) represents an unnamed value; and \(\ldots\) is used when matching inside lists, sets, and multisets.

**Cells:** K cells are defined explicitly in K modules using the `cell` keyword. A cell definition provides both the name of a cell and the contents:

```plaintext
cell nextLoc : Nat.
cell store : KMap{Location,Value}.
```

**Contexts:** Once added, strictness attributes apply to all occurrences of an operator. Sometimes, however, it makes more sense to apply strictness only in a certain context, such as when a construct is used with certain sorts. This functionality is provided using the `kcxt` keyword, which includes the attributes to apply to an operator given the context in which it appears. Note that the variable name, not just the position, can be used in the `strict` declaration.

```plaintext
kcxt deref(K) := K' [strict(K)].
```

**Configurations:** To indicate the final configuration that a language will use, a configuration declaration can be included using the `kconf` keyword. This declaration includes the nested K cell structure, with variables indicating the sorts inside each cell. Cells with an * character, such as `<thread*>`, can occur multiple times. The information given in a configuration declaration is used by the context transformers to automatically adapt any rules and equations to the final language configuration.

```plaintext
kconf <T> <thread*> <control> <k> K </k> </control> </thread*>
<state> Sigma </state> </T> <result> V </result>.
```
5.2 Module Examples

Using the constructs introduced in Section 5.1, this section provides a number of examples of different types of modules that can be created. Section 5.3 provides a complete example using a version of the IMP language.

5.2.1 Semantic Entities

Semantic entities in K definitions include configuration items, such as environments and stores, and sorts or operations used during computations, such as computation items and values. A simple example of a semantic entity definition is provided in the module INT. INT imports two modules, one provided in the K prelude (identified by the K/ prefix on the path), VALUE and K/INT. It then makes Int a subsort of Value, indicating that integers are considered values in a language that imports this module:

```k
module INT is
  imports VALUE, K/INT-SORT .
begin
  subsort Int < Value .
endmodule
```

A second example is module ENV. This shows the definition of an environment, which provides a mapping from names to locations (a store then maps locations to values; the separation easily allows features like nested scopes and reference parameters for functions). Like in module INT, existing K prelude definitions are imported, here because sorts Name and Loc are needed inside the module body definitions:

```k
module ENV is
  imports K/NAME-SORT, K/LOC-SORT .
begin
  sortalias Env = KMap{Name,Loc} .
  varprefix Env : Env .
  cell env : Env .
endmodule
```

K provides lists, multisets, and maps by default, allowing the definition to refer to maps from sort Name to sort Loc. The sortalias construct provides a way to give a more intuitive name to this map – Env – which can then be used in the remainder of the definition (including in any modules that import ENV).

To simplify definitions that use environments, the varprefix declaration allows the definition of a variable prefix. In any modules importing this module, any variables starting with Env and not given explicit definitions (or matching another, more specific prefix) will automatically be given sort Env (e.g., variables Env, Env8, Env', etc.).
Finally, the cell definition defines a K cell named \texttt{env}. This cell is given a sort \texttt{Env}, meaning it will hold items of sort \texttt{Env} (i.e., it will hold mappings from names to locations). This K cell can be used in parts of the definition that import this cell, including inside K-style rules and equations and in program configurations.

### 5.2.2 Abstract Syntax

Before defining the semantics of language constructs, the abstract syntax of those constructs needs to be defined. This is done using abstract syntax modules, which are defined using a tag of \texttt{[SYNTAX]} after the module name. A first example of an abstract syntax module is the syntax for arithmetic expressions, shown in module \texttt{EXP/AEXP[SYNTAX]}:

```kmetal
module EXP/AEXP[SYNTAX]
begin
    ### xsort AExp . subsort AExp < Exp .
    varprefix AE : AExp .
endmodule
```

One way to define the sort of arithmetic expressions would be to define a new sort which could be made a subsort of \texttt{Exp}, illustrated in a comment in the module. Here, to illustrate the use of sort renaming, the sort \texttt{Exp} is renamed to \texttt{AExp} using a sort renaming directive on the import of module \texttt{EXP}. A var prefix to refer to arithmetic expressions is then defined, allowing variables starting with \texttt{AE} to be automatically treated as having sort \texttt{AExp}.

A second abstract syntax module, defining the addition construct, is shown in module \texttt{EXP/AEXP/PLUS[SYNTAX]}:

```kmetal
module EXP/AEXP/PLUS[SYNTAX]
    imports EXP/AEXP[SYNTAX] .
begin
endmodule
```

Syntax is defined using mixfix notation with an algebraic notation similar to that used in Maude; in this case, the operator is defined using \texttt{xop} to indicate that it represents part of the abstract syntax for a language. To increase reusability of the module, it is recommended that each module define only one language construct, although it is possible to define multiple constructs in the same module.
5.2.3  Semantic Rules

Once the syntax has been defined, the semantics of each construct need to be defined as well. One explicit goal of the module system is to allow different semantics to be easily defined for each language construct. For instance, it should be possible to define a standard dynamic/execution semantics, a static/typing semantics, and potentially other semantics manipulating different notions of value (for instance, various notions of abstract value used during analysis). An initial example of a semantics module is a module to define a standard dynamic semantics for integer plus:

```
module EXP/AEXP/PLUS[DYNAMIC]
  imports EXP/AEXP/PLUS[SYNTAX] *
  (op _+_ now strict, extends + [Int * Int -> Int] ) .
begin
endmodule
```

Normally a semantics module will implicitly import the related syntax module. Here, since the import also modifies the attributes on an imported operator, the syntax module must be explicitly imported. Two attributes are modified. First, a strict attribute is added to note that the operator is now strict in all arguments, which will automatically generate the structural heating and cooling equations. Second, extends is used to automatically “hook” the semantics of the feature to the builtin definition of integer addition. This completely defines integer addition in the language, so no rules are needed.

A more typical semantics module will include rules:

```
module EXP/AEXP/PLUS[STATIC] is
  imports EXP/AEXP/PLUS[SYNTAX] with { op _+_ now strict } .
  imports TYPES .
begin
  vars T T' : Type .

  krl <k>[[int + int ==> int]] ...</k> .

  krl <k>[[T + T' ==> fail]] ...</k> [owise] .
endmodule
```

This module shows semantics for the same feature, but this time the static semantics (for type checking) are defined. Like in the dynamic semantics, the operator for plus is changed to be strict. In this case, though, the values being manipulated are types, not integers, so we also need to import the types and use them in the two rules shown, both of which use an ASCII version of the K notation. Reductions are shown inside semantic-style brackets ([[ and ]]), with an arrow (==>) dividing the before and after parts of the reduction.
Here, the first rule is for when an expression is type correct: the two operands are both integers, so the result of adding them is also an integer. If one of the operands is not an integer (the otherwise case), the rule will cause a type called fail, representing a type error, to propagate.

The next module shows the dynamic semantics of blocks:

```plaintext
module STMT/BLOCK[DYNAMIC] is
    imports STMT[SYNTAX], K/K, ENV.
begin
    var S : Stmt.

    krl <k> [[ begin S end ==> S -> restoreEnv(Env) ...<k>
        <env> Env </env>.
    endmodule
```

Here, no changes are made to the imported syntax, so there is no need to import the STMT/BLOCK[SYNTAX] module explicitly. In this language, blocks provide for nested scoping, so we want to ensure that the current environment is restored after the code inside the block executes. This is done by capturing the current environment, Env, and placing it on the computation in a `restoreEnv` computation item. The rule for `restoreEnv`, not shown here, will replace the current environment with its saved environment when it becomes the first item in the computation.

### 5.2.4 Language Definitions

Once the semantic entities, abstract syntax, and language semantics have been defined, they can be assembled into a language module, tagged `LANGUAGE`. An example is shown in Figure 5.2. The modules used to form the semantics are imported using `imports`, with the second and third `imports` given a tag. This tag will automatically be added to the path for each module, providing a more concise way to write the paths. The line `kconf` defines the language configuration used for IMP, including a store, an environment, a next location counter, and the main computation. Next, the `[[ ]]` operator initializes the configuration, given an initial computation (K) representing the program to run.

### 5.3 An Extended Example: Creating Language Extensions

The modularity features of K should allow defined language features to be reused in new languages and in extensions to existing languages. To illustrate this process using the K module system, an example language definition, for the

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3 An alternative would be to issue an error message and return the expected type in the hope of finding additional errors.
Kernel-C language \[158\], is shown. This is then extended to support exceptions, providing a typical example of how a language could be extended.

5.3.1 The Kernel-C Language

A number of modules make up the definition of the abstract syntax, configuration, and dynamic semantics for Kernel-C. These modules are discussed below.

Configurations

Kernel-C makes use of configuration items, such as Top, provided in the K prelude. It also uses four configuration items specific to Kernel-C. The first is the mem cell, created with a sort alias mapping sort Mem to maps from K to K (sort K is the general sort representing computations, and includes both unevaluated parts of the computation and computed results). Mem is used to represent the store:

\[
\text{module KERNELC/CONFIG/MEM is} \\
\text{import K/K.} \\
\text{begin} \\
\text{sortalias Mem = KMap\{K,K\}.} \\
\text{cell mem: Mem.} \\
\text{endmodule}
\]

The ptr cell is defined similarly, and is used to track pointer allocation and deallocation. Note that we could instead define a general map, KMap, from K to K, and then define individual cells over this map. Using different sort aliases provides a way to indicate the type of information held in the cell directly in the cell definition.

\[
\text{module KERNELC/CONFIG/PTRMAP is}
\]

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import K/K .
begin
  sortalias PtrMap = KMap(K,K) .
  cell ptr: PtrMap .
endmodule

Environments are also defined similarly, as maps from K to K, and provide mappings from names in the program to memory locations:

module KERNELC/CONFIG/ENV is
  import K/K .
begin
  sortalias Env = KMap(K,K) .
  cell env: Env .
endmodule

Finally, the out cell, representing output, is defined to just hold a single output stream (updated with each output operation), also represented as a K:

module KERNELC/CONFIG/OUT is
  import K/K .
begin
  cell out: K .
endmodule

Abstract Syntax

Kernel-C includes syntax similar to the C language. Expressions, defined in module KERNELC/EXP[SYNTAX], include generic expressions (arithmetic, boolean, logical) from the K prelude, as well as strings:

module KERNELC/EXP[SYNTAX] is
  import K/GENERIC-EXP-K-SYNTAX .
  import K/STRING-K-SYNTAX .
endmodule

The DEREF syntax module defines syntax for pointer dereferencing. Note that dereferencing is strict, since the expression indicating what is being dereferenced should be evaluated first:

module KERNELC/EXP/MEM/DEREF[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
endmodule

Module MALLOC then provides syntax for the Kernel-C malloc expression, which is similar to the malloc call found in C. malloc is also strict, with the expression indicating the amount of storage to allocate:

module KERNELC/EXP/MEM/MALLOC[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
endmodule
Also like in C, Kernel-C includes syntax for *free*’ing allocated memory, with `strict` indicating that the expression indicating the target of the `free` should be evaluated first:

```
module KERNELC/EXP/MEM/FREE[SYNTAX] is
    import KERNELC/EXP[SYNTAX] .
begin
endmodule
```

Like C, Kernel-C includes a ternary expression. The first position is strict, since it needs to be evaluated first to determine whether to evaluate the second or third expression next:

```
module KERNELC/EXP/TERNARY[SYNTAX] is
    import KERNELC/EXP[SYNTAX] .
begin
    xop _?_:_ : Exp Exp Exp -> Exp [strict(1)] .
endmodule
```

The syntax for boolean `not` is also renamed, this time equationally, turning it into a ternary expression. This ensures that separate semantics do not need to be given for `not` (hence the `aux` attribute):

```
module KERNELC/EXP/BOOL/NOT[SYNTAX] is
    import KERNELC/EXP[SYNTAX] .
    import KERNELC/EXP/TERNARY[SYNTAX] .
begin
    var E : Exp .
    eq ! E = E ? 0 : 1 .
endmodule
```

Assignment is defined as strict in the second argument, ensuring that the value to assign is computed before the assignment. As will be seen later, it is not strict in the first argument since behavior varies based on which language construct is being assigned into:

```
module KERNELC/EXP/ASSIGN[SYNTAX] is
    import KERNELC/EXP[SYNTAX] .
begin
    xop _:=_ : Exp Exp -> Exp [strict(2)] .
endmodule
```

The syntax for `and`, like that for `not`, is equationally transformed to use the ternary operator:

```
module KERNELC/EXP/BOOL/AND[SYNTAX] is
    import KERNELC/EXP[SYNTAX] .
    import KERNELC/EXP/TERNARY[SYNTAX] .
begin
    vars E E' : Exp .
    eq E && E' = E ? E' : 0 .
endmodule
```
The syntax for **or** is transformed in the same way, again allowing a semantics to be given by translation:

```plaintext
module KERNELC/EXP/BOOL/OR[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
  import KERNELC/EXP/TERNARY[SYNTAX] .
begin
  vars E E' : Exp .
  eq E || E' = E ? 1 : E' .
endmodule
```

The syntax given for **printf** is reminiscent of that used in C for printing integers. Note that only one integer can be printed at a time:

```plaintext
module KERNELC/EXP/PRINTF[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
endmodule
```

The **null** keyword is introduced so that it can be used in memory operations. Like in many C implementations, **null** is treated identically to 0:

```plaintext
module KERNELC/EXP/NULL[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
  eq null = 0 .
endmodule
```

After defining the syntax of expressions, the syntax for Kernel-C statements is given. Module **STMT** defines both statements and lists of statements, with a **renameTo** attribute indicating that lists of statements are turned into computations automatically:

```plaintext
module KERNELC/STMT[SYNTAX] is
begin
  xsorts Stmt StmtList .
  subsort Stmt < StmtList .
  xop __ : StmtList StmtList -> StmtList [renameTo _->_] .
endmodule
```

Expressions can be made into statements by following them with a semicolon:

```plaintext
module KERNELC/STMT/EXPSTMT[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
  import KERNELC/STMT[SYNTAX] .
begin
endmodule
```

An empty statement, represented by just a semicolon, is also available, like in C. It is renamed to the K identity, meaning it has the same semantics as doing nothing:
Two forms of blocks are provided, one with statements and an empty block. Currently Kernel-C does not provide nested scopes, so a block is automatically turned into a regular statement, in effect discarding the block brackets:

Conditionals are defined to support both one-armed and two-armed varieties. An equation automatically converts a one-armed conditional into a two-armed conditional with an empty block, which is why the first (one-armed) version is marked as aux (no equations should be generated for it, since it will be transformed anyway). The if statement is then strict in the first argument, ensuring the guard is evaluated before a branch is selected:

The while loop in Kernel-C looks the same as the equivalent C construct:

Finally, with all the other syntax defined, it is possible to define programs. Given in a form that looks very close to a simple C program, programs take a list of statements. The extraneous syntax is discarded, leaving just this statement list to execute:
5.3.2 Semantics

While defining the syntax of Kernel-C a number of constructs where transformed into other constructs, using either the renameTo attribute or using equations. Because of this, fewer semantic definitions are needed, with one definition often providing semantics for multiple (syntax-level) language features. Some semantic definitions are also given in the K prelude – these are brought in in module KERNELC/EXP[DYNAMIC] by importing module K/GENERIC-EXP-SEMANTICS:

```k
module KERNELC/EXP[DYNAMIC] is
  import K/GENERIC-EXP-SEMANTICS .
endmodule
```

The semantics for booleans automatically transforms them into numbers, similar to the semantics for operations that work with “booleans” in C (booleans in quotes, because C has no real booleans):

```k
module KERNELC/EXP/BOOL[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
begin
  eq #(true) = #(1) .
  eq #(false) = #(0) .
endmodule
```

The semantics for dereferencing use several K constructs. The context definition enforces strictness in assignment for the first operand when it is a dereferencing operation; the two keq definitions then give the semantics both for a lookup through a pointer and an assignment through a pointer:

```k
module KERNELC/EXP/MEM/DEREF[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
  import KERNELC/CONFIG/MEM .
  import KCONFIG/CONFIG .
begin
  vars K1 K2 : K .
  var N : Nat .
  vars V V' : KResult .
  kcxt * K1 := K2 [strict(K1)] .
  krl <k> [\#(N) ==> V] ...</k> <mem>... #(N) |-> V ...</mem> .
  krl <k> [\#(N) := V ==> V] ...</k> <mem>... #(N) |-> [V' ==> V] ...</mem> .
endmodule
```

The malloc semantics handle memory allocation, returning the new memory location and performing the proper bookkeeping in the pointer map stored in cell ptr. Note the use of K/FRESH-ITEM{K} below: this is a parameterized module from the K prelude (hence the prefix of K/), with the parameter the sort K:

```k
module KERNELC/EXP/MEM/MALLOC[DYNAMIC] is
```

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import KERNELC/EXP[DYNAMIC] .
import KERNELC/CONFIG/MEM .
import KERNELC/CONFIG/PTRMAP .
import KCONFIG/CONFIG .
import K/FRESH-ITEM{K} .

begin
  vars N N' : Nat . var NV : NatVar .

  op alloc : Nat Nat -> PtrMap .
eq alloc(N, 0) = .empty .
eq alloc(N, s(N')) = (#(N) |-> #(0)) &' alloc(N + 1, N') .

  krl <k> [[ malloc(#(N)) ==> #(N')]] ...</k>
  <ptr>... [[.empty ==> #(N') |-> #(N)]] ...</ptr>
  <nextItem> [[item(N') ==> item(N') + N]] </nextItem>
  <mem>... [[.empty ==> alloc(N', N)]] ...</mem> .
endmodule

Using \texttt{free} does the opposite, using this earlier bookkeeping information to properly clean up allocated storage:

module KERNELC/EXP/MEM/FREE[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
  import KCONFIG/CONFIG .
  import KERNELC/CONFIG/MEM .
  import KERNELC/CONFIG/PTRMAP .

  begin
    vars N N' : Nat .
    var V : KResult .
    var Mem : Mem .

    op void : -> KResult .
    op freeMem : PtrMap Nat Nat -> PtrMap .
eq freeMem(Mem, N, 0) = Mem .
eq freeMem((Mem &' (#(N) |-> V)), N, s(N')) = freeMem(Mem,N + 1,N') .
  krl <k> [[free(#(N)) ==> void]] ...</k>
  <ptr>... [[#(N) |-> #(N') ==> .empty]] ...</ptr>
  <mem>[[Mem ==> freeMem(Mem, N, N')]] </mem> .
endmodule

The ternary expression works the same as in C. The first expression is evaluated to a numeric value. If this value is not 0, it represents the true case, and the second expression is evaluated. If this value is 0, it represents the false case, and the third expression is evaluated:

module KERNELC/EXP/TERNARY[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .

  begin
    vars K1 K2 : K .
    var I : Int .

    eq (#(0)) ? K1 : K2 = K2 .
    ceq (#(I)) ? K1 : K2 = K1 if I neq 0 .
endmodule
Assignment works as expected; since this assignment does not involve dereferencing, the value can be stored directly in the environment, with storage to memory limited to those cases where the value can be updated through a pointer:

```kerneldyn```
module KERNELC/EXP/ASSIGN[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
  import KERNELC/CONFIG/ENV .
  import KCONFIG/CONFIG .
begin
  var X : Name .
  var V : KResult .
  var Env : Env .

  krl <k> [[X := V ==> V]] ...</k>
  <env> [[Env ==> Env[X <- V]]] </env> .
endmodule
```

Non-pointer lookups retrieve the value stored in the environment using assignment:

```kerneldyn```
module KERNELC/EXP/LOOKUP[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
  import KERNELC/CONFIG/ENV .
  import KCONFIG/CONFIG .
begin
  var X : Name .
  var V : KResult .

  krl <k> [[X ==> V]] ...</k>
  <env>... X |-> V ...</env> .
endmodule
```

The `printf` expression uses the given value to update the contents of the stream stored in cell `out`:

```kerneldyn```
module KERNELC/EXP/PRINTF[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
  import KCONFIG/CONFIG .
  import KERNELC/CONFIG/OUT .
begin
  var I : Int .
  var S : String .

  op stream : String -> K .

  krl <k> [[printf("%d",#(I)) ==> void]] ...</k>
  <out> [[stream(S) ==> stream(S + string(I,10)+ " ")]] </out> .
endmodule
```

The base semantics for statements import the base expression semantics, which they rely on:

```kerneldyn```
module KERNELC/STMT[DYNAMIC] is
  import KERNELC/EXP[DYNAMIC] .
endmodule
Expression statements can then be treated essentially as no-ops, once the expression is evaluated (which is automatic because of the strictness annotation on the `_;` operator). The value is not used, so it is just discarded:

```plaintext
module KERNELC/STMT/EXPSTMT[DYNAMIC] is
    import KERNELC/STMT[DYNAMIC] .
begin
    var V : KResult .
    eq V ; = .K .
endmodule
```

The `if` semantics also rely on strictness, with one rule each for the situation where the condition is false (i.e., 0) and where the condition is true (i.e., not 0):

```plaintext
module KERNELC/STMT/IF[DYNAMIC] is
    import KERNELC/STMT[DYNAMIC] .
begin
    vars K1 K2 : K .
    var I : Int .
    eq if (#(0)) K1 else K2 = K2 .
    ceq if (#(I)) K1 else K2 = K1 if I neq 0 .
endmodule
```

While loops are given a semantics in a standard way, with a translation into an equivalent conditional that first checks the while guard and then, if true, executes the body and then schedules the loop for another iteration:

```plaintext
module KERNELC/STMT/LOOP/WHILE[DYNAMIC] is
    import KERNELC/STMT[DYNAMIC] .
    import KERNELC/STMT/IF[DYNAMIC] .
begin
    vars K1 K2 : K .
    keq <k> [[while (K1) K2 ==> if (K1) (K2 -> while(K1) K2) else .K]] ...</k> .
endmodule
```

### 5.3.3 Language Definition

Using the already-defined modules, the language can be constructed. Dynamic semantics modules are imported, as well as syntax modules that are not imported by the dynamic semantics (remember, syntax modules are automatically imported into other modules with the same name, sans tag). The configuration is then created – it is not really needed here, since all cells are at the same level, but will come in handy later, and also provides good documentation. Finally, a run operation is provided, which, given an initial computation, will correctly set up the configuration so program execution can start:

```plaintext
module KERNELC/STANDARD[LANGUAGE] is
    import[Dynamic]
```
5.3.4 Adding Exceptions to Kernel-C

The exceptions extension to Kernel-C provides a simple exceptions mechanism, with a `try/catch` block similar to that used in Java and a `throw` statement that triggers an exception. The first step to adding exceptions is to define the configuration needed for exceptions, as well as some operators that work with the exception state:

```kernelc
begin
  var P : Pgm.
  var K : K.
  var Env : Env.
  var Mem : Mem.
  var Ptr : PtrMap.
  var I : Item.
  var S : K.

  kconf <T> <k> K </k> <env> Env </env> <mem> Mem </mem>
      <ptr> Ptr </ptr> <nextItem> I </nextItem>
      <out> S </out> </T>.

  op run : Pgm -> Config.
  eq run(P)
    = <T>
        <k> mkK(P) </k> <env>.empty </env>
        <mem>.empty </mem> <ptr>.empty </ptr>
        <nextItem> item(1) </nextItem>
        <out> stream("") </out>
    </T>.
endmodule
```

```kernelc
module KERNELC/CONFIG/EXCEPTIONS is
  import KERNELC/CONFIG/ENV.
  import K/K.
begin
  sortalias ExStack = KList{K}.

  cell es : ExStack.

  vars K K' : K.
  var ES : ExStack.

  op removeHandler : -> K.
  krl <k> [[ removeHandler ==> .K ]] ...</k>
      <es> [[ K, ES ==> ES ]] </es>.

endmodule
```
A new sort alias, ExStack, is declared. An exception stack is a list of computations (Ks) treated as a stack. A new cell, es, is also defined; this cell will be used in the semantics to hold the stack. Using this new sort alias and cell, three operators are then defined, along with semantic equations. removeHandler is used to remove the top exception handler from the stack without actually triggering the handler; handleException does the same, but it does trigger the exception handler, replacing the current computation (K’) with the handler computation. addHandler adds the handler onto the stack.

It is now possible to define the syntax and semantics for exceptions. The syntax for try/catch provides two statements: the code being handled, and the handler for when exceptions are thrown. try/catch is itself also a statement:

The semantics use the addHandler and removeHandler operators defined above. When a try/catch block is encountered, the semantics are defined to add a handler to the stack. This handler will run the computation in the catch block and will then pick up with whatever computation is after the try/catch construct. The code inside the try is then run after the handler is added. After this, the removeHandler item will remove the added handler; this will only be run if the handler is not triggered by an exception.

The syntax for throw defines it as a statement. Unlike in Java, it does not actually throw a value, so it is not given an expression to evaluate:
Finally, semantics are given to `throw`. A `throw` will trigger the exception handler using the `handleException` item:

```kern
module KERNELC/STMT/THROW[DYNAMIC] is
  import KERNELC/STMT[DYNAMIC] .
  import KERNELC/CONFIG/EXCEPTIONS .
begin
  krl <k> [[ throw; ==> handleException ]] ... </k>
endmodule
```

The new features are added by creating a new language version with imports for the new modules. Note that none of the old modules needed to change. Here we add the exception stack to the configuration, and we also add an additional level of grouping, with the `k` and `es` cells added under another cell, `control` (which we assume, for this example, is imported from the prelude, but could easily be added manually). The use of context transformers allows the existing rules that use `k` and other cells to continue to function unchanged.

```kern
module KERNELC/WEXCEPTIONS[LANGUAGE] is
  import[DYNAMIC]
    KERNELC/EXP/BOOL, KERNELC/EXP/MEM/DEREF, KERNELC/EXP/MEM/MALLOC,
    KERNELC/EXP/MEM/FREE, KERNELC/EXP/TERNARY, KERNELC/EXP/ASSIGN,
    KERNELC/EXP/LOOKUP, KERNELC/EXP/PRINTF, KERNELC/STMT/EXPSTMT,
    KERNELC/STMT/IF, KERNELC/STMT/LOOP/WHILE, KERNELC/STMT/TRYCATCH,
    KERNELC/STMT/THROW .
  import[SYNTAX]
    KERNELC/EXP/TERNARY, KERNELC/EXP/BOOL/NOT, KERNELC/EXP/BOOL/AND,
    KERNELC/EXP/BOOL/OR, KERNELC/EXP/NULL, KERNELC/STMT/EMPTY,
    KERNELC/STMT/BLOCK, KERNELC/PGM .
begin
  var P : Pgm .
  var K : K .
  var Env : Env .
  var Mem : Mem .
  var Ptr : PtrMap .
  var I : Item .
  var S : K .
  var ES : ExStack .

  kconf <T> <ctrl> <k> K </k> <es> ES </es> </control>
    <env> Env </env> <mem> Mem </mem>
    <ptr> Ptr </ptr> <nextItem> I </nextItem>
    <out> S </out> </T> .

  op run : Pgm -> Config .
  eq run(P) = <T>
    <control> <k> mkK(P) </k> <es> .empty </es> </control>
    <env> .empty </env> <mem> .empty </mem>
    <ptr> .empty </ptr> <nextItem> item(1) </nextItem>
```
5.4 Translating K Modules to Maude

K module definitions are translated into a K Maude format supported by the current release of the K tools, which is then translated to Maude. This section focused on the first part of this translation; the second is part of ongoing work, but is not part of the work on the module system. The K Maude format used in the K tools is not modular, allowing either one module, with the entire language definition, or three, one for syntax, one for the configuration, and one for semantics. Because of this, one of the main tasks of the translation is to “flatten” the information given in the K modules into a non-modular form.

Standard Features: Many of the constructs in the module system are already valid in the K tools format, and require no translation. This includes subsorts and operator definitions. Other constructs have very lightweight translations: syntax ops are translated into regular ops with the same names, signatures, etc. One current limitation of the module system tools support is that, because the parsing is currently deferred until the K tools form of the semantics is presented to Maude, several features that rely on parsing are not yet fully supported. This includes altering of attributes on imports, some renamings, and some uses of exports and requires.

Features Requiring Significant Translation: The remaining features are handled as follows:

- Each sort is defined in its own module, with that module then imported into the module that initially defined the sort. A view is created automatically for each sort from TRIV to the sort module, mapping Elt to the sort.
- Each sort alias is defined in its own module, using import renamings to map the original name for the sort (e.g., Map\{K,K\}) to the alias name (e.g., Mem).
- Syntax sorts, syntax operators, and subsorts using the syntax sorts are all generated into the syntax module: the sorts are added to the module as imports, while the operators and subsorts are added as actual operator and subsort definitions.
- Cells are translated into operators in the configuration module, with a special attribute on each operator describing the cell contents.
- Other items are translated into a single semantics module. Variables and variable prefixes are used to decorate variables in rules and equations with
sort information, and are not otherwise included in the final generated module. Variable prefixes are used based on reachability, to ensure that only imported (directly or indirectly) prefixes can be used. Configurations and contexts (with variable sort decoration) are also brought over directly. Semantic sorts and sort aliases are handled identically to syntax sorts and sort aliases.

**Module Headers:** Exports clauses that do not mention syntax items (the standard case – generally one does not hide abstract syntax, but instead hides special operators or sorts used inside other modules) are modeled by moving the module contents to a second module. The original module then imports this second module, renaming all private operators, sorts, and cells. This does not prevent use of these private constructs, but protects against inadvertent use or duplication, which is the goal of the exports feature.

Imports are converted directly into Maude-style including directives, with any special prefixes (e.g., the \texttt{K/INT-SYNTAX}) removed first. These import paths are collected based on which modules are reachable, ensuring all needed imports are given in the proper three modules while not either importing modules that have been translated away or that only contain information used by another of the three modules (e.g., the syntax does not import all the semantics).

### 5.5 The Online Semantics Repository

One goal of the K module system is to allow modules to easily be shared and reused. To help support this goal an online repository of semantics modules is being developed. At this point, a basic version of the repository, targeted specifically at K modules, has been developed. It is expected that further work on this repository will include closer tool integration and the ability to support additional module formats, such as standard rewriting logic semantics modules and MSOS modules.

#### 5.5.1 The Repository Backend

The repository currently uses a database to store information about each module. Each module record is assigned a unique identifier, and also contains information such as: module name; module version; a description of the module; when the module was last updated, and by whom; the type of semantics inside the module; the modules required for this module to work properly; and the features defined by this module. The current main purpose of the backend is to store and provide data for the module exchange format, described below, but in the future tools may directly access the database to work with the modules in the repository.
5.5.2 An XML Format for Module Exchange

To enable the exchange of semantic modules between K tool support and the module repository, and with an eye towards future interfacing with other tools or other module formats, an initial definition of a shared format for exchanging semantic modules has been defined. The XML Schema definition for this format can be found in Figure 5.3.

An XML document containing a semantic module definition is made up of the overall definition, a list of other required modules, and a list of features provided by this module. Figure 5.4 focuses on the overall definition. It includes the module name, a module namespace, providing a way to group modules by provider, and a version number for the module. Also included are several fields indicating when the module was last modified (lastModifiedOn), and by whom (lastModifiedBy); the type of semantics (semanticsType), such as K or MSOS; and a description of the module.

The requirements of the module, currently in field requirements, are given in terms of required modules. Each required module is of type RequiredModule, the definition of which is given in Figure 5.5. The information for the module requirement includes a path, which is made up of the namespace followed by the module name, and a required version. Currently the required version must be an exact match; a possible future modification is to allow additional information, either with version numbering patterns or comparison operations (ranges, version greater than the given number, etc).

Individual features, of type Feature, are given inside the features tag. The format for features is shown in Figure 5.6. A feature description is made up of a feature name, a feature description, a numeric feature type, and the feature contents. The meaning of these fields is dependent on the style of semantics. In K, the name and description provide optional descriptive information, the contents is the actual K text, and the type indicates the type of feature being given, with different numbers for syntactic operators, k rules, etc.

Figure 5.7 provides an example of a module that defines both the abstract syntax and semantics for addition. The name of the module is Exp/AExp/Plus, and it is part of namespace FSL. The version number is set to 1.0, and the semantics type to K. Two modules are listed as requirements, both in the FSL namespace, and both at version 1.0 as well. Also, two features are defined. The first is a syntax operator, the second a K rule for addition. The name and description are arbitrary, while the types indicate the type of feature (6 is a syntax operator, 14 a K rule).

5.5.3 Client-Side Tool Support

Client-side tool support is based around a command line tool that communicates with the repository using web services. Currently, several operations are supported:
<?xml version="1.0" encoding="UTF-8"?>
  targetNamespace="http://fsl.cs.uiuc.edu/Schema"
  elementFormDefault="qualified" attributeFormDefault="unqualified">
  <xs:element name="semanticModule" type="SemanticDefinition">
    <xs:annotation>
      <xs:documentation>The definition of a single semantics module.</xs:documentation>
    </xs:annotation>
    <xs:complexType name="SemanticDefinition">
      <xs:sequence>
        <xs:element name="name" type="xs:normalizedString"/>
        <xs:element name="namespace" type="xs:normalizedString"/>
        <xs:element name="version" default="1.0">
          <xs:simpleType>
            <xs:restriction base="xs:decimal">
              <xs:fractionDigits value="3"/>
              <xs:totalDigits value="9"/>
              <xs:minExclusive value="0"/>
            </xs:restriction>
          </xs:simpleType>
        </xs:element>
        <xs:element name="lastModifiedBy" type="xs:string" minOccurs="0"/>
        <xs:element name="lastModifiedOn" type="xs:dateTime" minOccurs="0"/>
        <xs:element name="semanticsType" type="xs:normalizedString"/>
        <xs:element name="description" type="xs:string"/>
        <xs:element name="requirements">
          <xs:complexType>
            <xs:sequence>
              <xs:element name="requiredModule" type="RequiredModule" minOccurs="0" maxOccurs="unbounded"/>
            </xs:sequence>
          </xs:complexType>
        </xs:element>
        <xs:element name="features">
          <xs:complexType>
            <xs:sequence>
              <xs:element name="feature" type="Feature" minOccurs="0" maxOccurs="unbounded"/>
            </xs:sequence>
          </xs:complexType>
        </xs:element>
      </xs:sequence>
    </xs:complexType>
  </xs:element>
</xs:schema>

Figure 5.3: Module Exchange Format: Complete XML Schema
Figure 5.4: Module Exchange Format: Module Definition

Figure 5.5: Module Exchange Format: Required Modules

Figure 5.6: Module Exchange Format: Module Features
• retrieve a list of the modules contained in the repository;
• retrieve a specific module from the repository;
• retrieve a specific module from the repository, including all modules transitively required by the module.

Modules are currently inserted by generating SQL directly from the modtool command-line interface to the module system, which is also used to perform translations from modules in the module system into the Maude K tool format. One future task is to make the repository open for outside updates, which will require proper security support to ensure that only authorized users can modify repository contents.

5.6 Discussion

Chapters 3, 4, and 5 presented methods and tools for the modular definition of language features, including (in this chapter) a module system to allow the reuse of these features. This section discusses some of the limitations of this work; a comparison with other related work is provided in Chapter 9.

First, some language features seem to be inherently non-modular. While it is possible to define these features using K, it may still be necessary to either alter one or more imported modules (not recommended) or create new versions of

```xml
<?xml version="1.0" encoding="UTF-8"?>
<semanticModule xmlns="http://fsl.cs.uiuc.edu/Schema"
    xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
    xsi:schemaLocation="http://fsl.cs.uiuc.edu/Schema/Semantics.xsd">
    <name>Exp/AExp/Plus</name>
    <namespace>FSL</namespace>
    <version>1.0</version>
    <semanticsType>K</semanticsType>
    <description>Syntax and semantics of integer addition.</description>
    <requirements>
        <requiredModule>
            <version>1.0</version>
            <path>FSL::K/Builtins/Configs/K</path>
        </requiredModule>
    </requirements>
    <features>
        <feature>
            <name>Syntax</name>
            <description>Abstract syntax for plus.</description>
            <type>6</type>
            <contents>xop _+_ : AExp AExp -> AExp [strict].</contents>
        </feature>
        <feature>
            <name>Semantics</name>
            <description>Dynamic semantics for plus.</description>
            <type>14</type>
            <contents>krl <k> [[ I1 + I2 == int+(I1,I2) ]]...</k> .</contents>
        </feature>
    </features>
</semanticModule>
```

Figure 5.7: Module Exchange Format: Example Module
some modules that are specific to the feature being defined. One example of this, presented in Chapter 3, was synchronized methods. Because locks need to be properly tracked at method entry and exit, defining synchronized methods also required changing the definition of exceptions, ensuring that any locks acquired on entry to a synchronized method are released as the stack frame is unrolled. Another example would be a Java-like `finally` clause on an exception handler, which impacts the semantics of method return and loop `break` and `continue` as well. For instance, if an exception handler is provided in the body of a loop, a `break` statement given inside the `try` block would trigger the `finally` clause. In both of these cases, this means the semantics of other features need to change when the new feature is introduced.

Second, while the K module system is quite flexible, the downside of this is that the module system does not provide support for good definitional style. In other words, it allows one to write modules that are not in fact modular, packing a number of features into a single module. While we believe that this is essential, to allow the user the freedom to define modules in a style with which the user is comfortable, it would be useful to allow the enforcement of good style in cases where the intent is to later share the modules with others.

Third, the support in the K module system of Maude attributes, such as the `format` and `prec` attributes, can tie K modules defined using the module system closely to Maude. For instance, the `prec` attribute could be added to the `DEREF` abstract syntax shown above to allow for the proper parsing of programs given as Maude terms:

```kmetal
module KERNELC/EXP/MEM/DEREF[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
endmodule
```

Or, the `format` attribute could be used in the abstract syntax for `MALLOC` to provide proper formatting (spacing, newlines, colors, etc) of the operator when it is output in Maude:

```kmetal
module KERNELC/EXP/MEM/MALLOC[SYNTAX] is
  import KERNELC/EXP[SYNTAX] .
begin
  xop malloc('(_') : Exp -> Exp [format (g b o b o) strict] .
endmodule
```

Since it is not required that one use these attributes, it is still possible to write modules for features which can then be reused in different languages (for instance, languages where the precedence is different). However, it is currently necessary to use these attributes if one wishes to provide programs directly as Maude terms, instead of writing a language front-end which will generate an abstract syntax for the program in a Maude-friendly form (such as is done in both KOOL and Beta). One possible solution is to add a layer in the transformation
from K modules to a target platform, such as Maude, that allows transformations – the addition of attributes, etc – to be performed on the individual items in a module body.
Chapter 6

Language Design and Performance

Many methods of defining languages are based solely on paper definitions, potentially with tool support to help properly typeset rules in the semantics. For such formalisms performance is not even a consideration. However with executable specifications, performance can be critical, especially when using the definitions in practical situations, such as during language prototyping or program analysis. This chapter discusses research on improving the performance of K definitions implemented using Maude. Section 6.1 focuses mainly on execution performance, illustrating how changes in the memory representations used by language definitions can speed up certain programs. Section 6.2 then shifts this focus to analysis performance, showing how changes to the KOOL language, discussed in Chapter 3, can both make analysis faster and allow larger programs to be analyzed. This chapter serves as a bridge between the earlier chapters, which have focused on language semantics, and the later chapters, which focus more directly on program analysis.

The definitions of syntax and semantics in this chapter are shown using Maude syntax, instead of directly in K syntax. All equations and rules use the K definitional style in Maude.

6.1 Execution Performance

Because of the executability of the semantic rules used in K definitions, changes to the definitions of language features and the semantic configuration can have a large impact on performance. One temptation is to modify features in the language directly, trying a number of permutations to determine which perform best. While this may be acceptable in certain circumstances, in other cases it may lead to definitions of language features that either no longer really define the feature correctly (for instance, by restricting behavior to improve performance, at the cost of eliminating behaviors allowed by the language) or are too complex, sacrificing simplicity and understandability for speed.

One promising direction is to change, not the rules defining the semantics of language features, but those defining the surrounding configuration. Since the configuration is seen as an abstraction inside the language rules, changes to the configuration can occur without requiring changes to the semantics of language
constructs. Beyond this, configurations are often reused between languages, allowing improvements to be directly leveraged inside other definitions. This section describes changes to two languages, SILF and KOOL, both at the level of the memory representation in the configuration. Section 6.1.1 shows the first memory representation change, introducing a stacked memory model to SILF; performance comparisons show the benefit of this model, while comparisons with the prior version show that few changes to the semantics are needed. After this, Section 6.1.2 shows the second change, the addition of a basic mark-sweep garbage collector to KOOL, including discussions of performance and reusability.

### 6.1.1 SILF and Stacked Memory

SILF, the Simple Imperative Language with Functions, is a basic imperative language with many core imperative features: functions, global variables, loops, conditionals, and arrays. Programs are made up of a series of variable and function declarations, with a designated function named `main` serving as the entry point to the program. The syntax of SILF is shown in Figure 6.1, while a sample program, which computes the factorial of 200 recursively, is shown in Figure 6.2.

The SILF Memory Model

In SILF, memory is allocated automatically for global and local variables and arrays, including for the formal parameters used in function calls. Users are not able to allocate additional storage with operations like `new` or function calls like C’s `malloc`, and are not able to create pointers/references or capture variable addresses. SILF includes a simple memory model, where memory is represented as a set of `Location × Value` pairs, referred to as `StoreCells`; the entire set is just called the `Store`. This, or something very similar, is the standard

---

1In SILF, a `StoreCell` is actually called a `<Location><Value>`, which has an associated `<Location><Value>Set` representing the `Store`. The terminology is changed here to make it
model used in a number of languages defined using K or the computation-based definitional style.\footnote{simpler to read and type.}

\begin{verbatim}
sorts StoreCell Store .
subsort StoreCell < Store .
op [_,_] : Location Value -> StoreCell .
op nil : -> Store .
op __ : Store Store -> Store [assoc comm id: nil] .
\end{verbatim}

The main advantage of this model is that memory operations are simple to define; memory update and lookup can be performed just using matching within the Store, as shown here for variable lookup:

\begin{verbatim}
eq k(exp(X) -> K) env(Env [X,L])
  = k(lookupLoc(L) -> K) env(Env [X,L]) .
eq k(lookupLoc(L) -> K) store(Mem [L,V])
  = k(val(V) -> K) store(Mem [L,V]) .
\end{verbatim}

Here, exp(X) means there is an expression X, a variable name; matching is used to find the location of X, L, in the current environment, a set of Name \times Location pairs. This triggers the lookup of location L using operation lookupLoc. When this operation is processed, matching is performed against the store, returning the value V stored at location L.

This model has a major disadvantage, though: old locations are never removed from the store, even when they become unreachable, which happens quite often (formals become unreachable after each function return, for instance). As memory grows, it takes longer to match against the store, slowing execution performance.\footnote{Newer definitions often use the built-in MAP module instead; this is used by KOOL, for instance, as described in Section \ref{sec:kool}.}

\begin{verbatim}
function factorial(n)
begin
  if n = 0 then
    return 1
  else
    return n * factorial(n - 1)
  fi
end

function main(void)
begin
  write factorial(200)
end
\end{verbatim}

Figure 6.2: Recursive Factorial, SILF
Stacked Memories

As mentioned above, it is not possible in SILF to dynamically allocate memory or take the addresses of variables. This prevents addresses from escaping a function, since there is no way to return a pointer to something inside the function; because of this, it should be possible to discard all memory allocated for the function call when the function returns. A conceptually simple way to do this is to change from a flat memory to a stack of memories, with the memory for the current function on top and the global memory on the bottom. Memories can still be treated as sets of StoreCells, but each set can be much smaller, containing just the cells allocated in the current function, and each set can easily be discarded simply by popping the stack at function return. Following this reasoning, the memory model for SILF can be changed appropriately:

```
sort StackFrame Stack .
subsort StackFrame < Stack .
op [_,_] : Nat Store -> StackFrame .
op nil : -> Stack .
op _,_ : Stack Stack -> Stack [assoc id: nil] .
```

Here, each element of the stack is referred to by the name StackFrame, a familiar term meant to show the intuition behind the technique. Each StackFrame is actually a pair, a Store and a natural number representing the first location in the frame; attempts to access a lower numbered location need to check in earlier frames, here the bottom frame, since SILF’s scoping only allows access to local or global names and does not allow nested functions. These StackFrames are assembled into Stacks, with the head of the list as the top element of the stack and the last element of the list the global frame.

Location lookup is now slightly more involved:

```
op stackLookup : Location Stack -> Value .
op lvsLookup : Location Store -> Value .

eq k(lookupLoc(L) -> K) store(ST)
  = k(val(stackLookup(L,ST)) -> K) store(ST) .

ceq stackLookup(loc(N),([Nb,Mem], ST))
  = lvsLookup(loc(N),Mem)
  if N >= Nb .

ceq stackLookup(loc(N),([Nb,Mem], ST, [Nb',Mem']))
```

This seems restrictive, but is actually standard for stack-allocated memory in imperative or object-oriented languages without address capture, such as Java or Pascal. Heap-allocated memory could not be similarly discarded.
Now, location lookup just triggers `stackLookup`, which has two equations representing the two cases mentioned above. If the location number \( N \) is at least \( Nb \), the smallest location in the stack, the location should be in the current stack frame. If the location number is smaller than \( Nb \), it must be the location of a global variable, which should be in the frame at the bottom of the stack. Both cases then use a helper, `lvsLookup`, to find location \( L \) inside the store in the appropriate frame, using matching to find the matching `StoreCell` and retrieve the value.

**Evaluation**

To evaluate the effectiveness of the stacked memory model versus the standard flat memory model, seven test cases were executed in SILF under both models. The test cases implemented several standard recursive and iterative algorithms, with the intent being to not bias the tests in favor of either recursive or iterative styles of programming. The test cases were:

- `factorial`, recursively calculating the factorial for 20, 40, ..., 180, 200;
- `factorial2`, same as `factorial`, but for 1 ... 200;
- `ifactorial`, an iterative version of `factorial`;
- `ifactorial2`, an iterative version of `factorial2`;
- `fibonacci`, a recursive algorithm computing the fibonacci numbers from 1 to 15;
- `qsort`, a quick sort of two arrays of 100 elements;
- `ssort`, a selection sort of two arrays of 100 elements.

In all cases, the total execution time, total number of rewrites, and rewrites per second were recorded. The performance results are shown in Figures 6.3 and 6.4.

The results indicate that the stacked memory model provides improved performance over the flat memory model in many different programs, including all those tested here. Based on the total rewrites it is clear that the stacked model in some sense does more work, which is needed to maintain the stack and look up memory locations at different levels. It is also clear, though, that it does the work much more quickly, shown in the Rewrites/sec column, illustrating the benefit to matching performance of keeping the store small. The cost for making the change is fairly low, as well, since changing to the stacked memory model
requires few changes to SILF. Beyond adding new sorts and operations to model having stacks of memory frames, it was only necessary to change 6 existing SILF equations – specifically, those equations already dealing with memory, or with function calls and returns.

6.1.2 KOOL and Garbage Collection

The KOOL memory representation is structured similarly to the default representation used by SILF, with two differences. First, instead of defining the store explicitly, it is defined using the built-in MAP module. Second, instead of just mapping locations to values, the stores maps locations to a sort ValueTuple, which contains the value as one of its projections:

\[
\text{protecting MAP\{Location,ValueTuple\} *}
\]

\[
\text{(sort Map\{Location,ValueTuple\} to Store) .}
\]

\[
\text{op \{\_,\_,\_\} : Value Nat Nat -> ValueTuple .}
\]

Lookups and updates then use the MAP-provided functionality, supplemented with some additional operations for adding more than one mapping to the Store at once and for extracting the value from the tuple.

Since KOOL is multi-threaded, memory accesses to shared locations can compete. In work on analysis performance \[84]\ described in Section 6.2 memory was segregated into memory pools, with a shared pool for locations accessible from multiple threads and a non-shared pool for locations accessible from only one thread. Currently, this is represented instead as one memory pool, with the first Nat flag in the ValueTuple indicating whether the location is shared. Based
on the setting of this flag, rules or equations are used to access or update values in the store. The logic for location lookup is shown below; similar equations and rules for location assignment are not shown. Here, \( L \) is a location in memory, \( N \) and \( M \) are natural numbers, \( V \) is a value, \( \text{Mem} \) is the store, and \( \text{CS}, \text{TS}, \) and \( \text{KS} \) represent other parts of the state that are not needed directly in the equations and rules:

\[
\begin{align*}
\text{op} \ llookup &: \text{Location} \to \text{ComputationItem} . \\
\text{op} \ slookup &: \text{Location} \to \text{ComputationItem} . \\
\text{op} \ \text{isShared} &: \text{Store Location} \to \text{Bool} . \\
\text{op} \ \text{getValue} &: \text{Store Location} \to \text{Value} . \\
\text{eq} \ \text{isShared}(_,\_,(L \mapsto [V,1,N], \text{Mem}), L) &= \text{true} . \\
\text{eq} \ \text{isShared}(\text{Mem}, L) &= \text{false} \ [\text{otherwise}] . \\
\text{ceq} \ \text{getValue}(\text{Mem},L) &= V \text{ if } [V,N,M] := \text{Mem}[L] . \\
\text{ceq} \ \text{threads}(t(\text{control}(k(llookup(L) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem}) \\
\quad = \text{threads}(t(\text{control}(k(\text{val(getValue(Mem,L)}) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem})) \\
\quad \text{if not isShared(Mem,L)}) . \\
\text{ceq} \ \text{threads}(t(\text{control}(k(llookup(L) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem}) \\
\quad = \text{threads}(t(\text{control}(k(slookup(L) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem}) \\
\quad \text{if isShared(Mem,L)}) . \\
\text{rl} \ \text{threads}(t(\text{control}(k(slookup(L) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem}) \\
\quad \to \text{threads}(t(\text{control}(k(\text{val(getValue(Mem,L)}) \to K) \text{CS} \text{TS} \text{KS}) \text{mem}(\text{Mem}) .
\end{align*}
\]

The first two equations define the \text{isShared} operation, which returns true when \( L \) is marked as shared (i.e., accessible by multiple threads) in \( \text{Mem} \). The third defines \text{getValue}, used for extracting the value at location \( L \) from the value tuple stored in \( \text{Mem} \). Following these definitions, the remaining two equations and one rule define the actual process of retrieving the value at location \( L \) from the store. In the first equation, \( L \) is not shared, so the value can be retrieved from \( \text{Mem} \) directly using \text{getValue}. In the second equation, \( L \) is shared; this causes lookup to switch over to a shared lookup operation. The rule then defines this shared lookup; the definition is identical to that for unshared locations, except in this case a rule is used, indicating that this could represent a race condition.

This model shares the same disadvantage as the original SILF model – old locations are never removed from the store, even when they become unreachable. And, since KOOL is a pure object-oriented language (boxing is not used here), locations become unreachable constantly. An expression such as \( 1 + 2 + 3 \) is syntactic sugar for \((1. + (2.)). + (3)\). New objects are created for the numbers 1, 2, 3, 3 again, and 6, with all but the last just temporaries that immediately become garbage. Unlike in SILF, a simple solution like stack frames cannot be used to remove unreachable objects, since often references to objects will be returned as method results. Without more sophisticated analysis, such as escape analysis \cite{149,17}, it must be assumed that any objects created in a method could escape,
meaning they cannot just be discarded on method exit.

Overall, the constant expansion of memory, the lack of obvious ways to reduce the memory size, and the performance decrease related to using a larger memory can make it difficult to run even some fairly small programs just using the semantics-based interpreter. Since one of the goals of defining KOOL is to allow for quick, easy experimentation with language features, a way to decrease the memory size and increase performance, without having to change language features in unwanted ways, is crucial.

Defining Garbage Collection

A solution common to object-oriented languages is to use garbage collection. Garbage collection fits well with KOOL’s allocation model, which uses new to create new objects but does not provide for explicit deallocation; it also accommodates the regular use of intermediate objects, which often quickly become garbage, in computations. If done properly, a GC-based solution also has the advantage that it can be defined at the level of the KOOL configuration, leaving the rules used to define language features unchanged.

The garbage collector defined below is a simple mark-sweep collector [96]. Mark-sweep collectors work by first finding a set of roots, which are references into the store. All locations transitively reachable from the roots are marked as being reachable (the marking phase); all unmarked locations are then removed from memory (the sweeping phase). GC equations can be divided into language-dependent equations, which need to be aware of language constructs, and language-independent equations, which just work over the structure of the memory and could be used in any language with the same Store definition.

Language-Independent Rules

The rules to mark and sweep memory locations during collection are separated into four phases. In the first phase, gcClearMem (seen in state item ingc), a flag on each memory location (the third element of the ValueTuple) is set to 0. By default, then, all memory locations are assumed to be unreachable at the start of collection. The state component used to hold the memory is also renamed, from mem to gcmem. This has the benefit of blocking other memory operations during collection without requiring the other operations to even be aware of the collector:

\[
\begin{align*}
\text{op } gcClearMem : & \rightarrow \text{GCState} . \\
\text{op } unmarkAll : & \text{Store } \rightarrow \text{Store} . \\
\text{eq mem(Mem) ingc(gcClearMem)} = & \text{gcmem(unmarkAll(Mem)) ingc(gcMarkRoots)} . \\
\text{eq unmarkAll(_’,_(L |\rightarrow [V,N,M], Mem))} & \\
\end{align*}
\]
In the second phase, \texttt{gcMarkRoots}, all locations directly referenced in computation portions of the KOOL state (inside the computation and in the stacks, for instance, but not in the memory) are found using \texttt{KStateLocs}, one of the language-dependent portions of the collector. Each of these root locations, stored in \texttt{LS}, is then marked by setting the third element of the \texttt{ValueTuple} at that location to 1:

\begin{verbatim}
  op gcMarkRoots : \rightarrow \textit{GCState} .
  op markLocsInSet : \textit{Store} \textit{LocationSet} \rightarrow \textit{Store} .
  op mark : \textit{Store} Location \rightarrow \textit{Store} .
  ceq threads(KS) ingc(gcMarkRoots ) gcmem(Mem )
    = threads(KS) ingc(gcMarkTrans(LS)) gcmem(markLocsInSet(Mem,LS))
    if LS := KStateLocs(KS) .
  eq markLocsInSet(Mem, (L LS)) = markLocsInSet(mark(Mem,L), LS) .
  eq markLocsInSet(Mem, emptyLS) = Mem .
  eq mark(_,_,(L |-> [V,N,M], Mem), L) = _,_,(L |-> [V,N,1], Mem) .
\end{verbatim}

Next, the third phase, \texttt{gcMarkTrans}, determines the locations reachable transitively through the root locations. It works using both the \texttt{iterate} and \texttt{unmarkedOnly} operations; the first determines the set of locations reachable in one step from a given set of locations (if an object at location \texttt{L} holds references to objects at locations \texttt{L1} and \texttt{L2}, \texttt{L1} and \texttt{L2} would be reachable in one step, but not any locations referenced by the objects at \texttt{L1} or \texttt{L2}), while the second filters this to only include locations that have not already been marked. At each iteration the locations found are marked and the process continues from just these newly-marked locations, ensuring that the traversal eventually terminates when no new, unmarked locations are found:

\begin{verbatim}
  op gcMarkTrans : LocationSet \rightarrow \textit{GCState} .
  op iterate : LocationSet \textit{Store} \rightarrow LocationSet .
  op unmarkedOnly : LocationSet \textit{Store} \rightarrow LocationSet .
  ceq threads(KS) ingc(gcMarkTrans(LS)) gcmem(Mem )
    = threads(KS) ingc(gcMarkTrans(LS')) gcmem(Mem')
    if LS' := iterate(LS,Mem) /\ LS' =/= emptyLS /\ Mem' := markLocsInSet(Mem,LS') .
  eq threads(KS) ingc(gcMarkTrans(LS)) gcmem(Mem)
    = threads(KS) ingc(gcSweep ) gcmem(Mem) [owise] .
\end{verbatim}
Finally, the fourth phase, gcSweep, uses the removeUnmarked operation to discard all memory locations not marked during the sweep performed in steps two and three. It also moves the store back into mem, so other parts of the semantics can again see the store:

\[
\text{op gcSweep : } \rightarrow \text{GCState} .
\]
\[
\text{eq ingc(gcSweep) gcmem(Mem) = ingc(noGC(0)) mem(removeUnmarked(Mem))}.
\]

**Language-Dependent Equations** Language-dependent equations are used to gather the set of roots from the computation and any other parts of the state (such as stacks) that may contain them. Traversal of the state is initiated using KStateLocs, which returns a set of all locations found in a given state. KStateLocs is defined inductively over the various state components, with other operations specifically designed to deal with computations, computation items, stacks, and other state components. Examples of the equations used to find the locations inside the method stack and the computation are shown below:

\[
\text{op KStateLocs : KState } \rightarrow \text{LocationSet} .
\]
\[
\text{eq KStateLocs(mstack(MSTL) CS) = MStackLocs(MSTL) KStateLocs(CS)}.
\]

\[
\text{op MStackLocs : MStackTupleList } \rightarrow \text{LocationSet} .
\]
\[
\text{eq MStackLocs ([K,CS,Env,oref(L),Xc], MSTL) = KLocs(K) KStateLocs(CS) ListToSet(envLocs(Env)) ListToSet(valLocs(oref(L))) MStackLocs(MSTL) .}
\]
\[
\text{eq MStackLocs(empty) = emptyLS}.
\]

\[
\text{op KLocs : Computation } \rightarrow \text{LocationSet} .
\]
\[
\text{eq KStateLocs(k(K) CS) = KLocs(K) KStateLocs(CS) .}
\]
\[
\text{eq KLocs(llookup(L) -> K) = L KLocs(K) .}
\]

KLocs deserves special comment, since it is the main operation that needs to be modified to account for new language features. KLocs is defined for each computation item in the language that may hold locations. This means that, when new computation items which can contain locations are added, the collector must be updated properly. A method of automatically transforming a theory into one with garbage collection would eliminate this potential source of errors, but has not yet been investigated.

Along with the equations used to find the roots, additional equations are used to find any locations referenced by a value (for instance, the locations referenced
in the fields of an object). These equations are then used when finding the set of locations reachable transitively from the root locations. In this case, it was possible to reuse equations developed in earlier work \[84\] that were used to find all locations reachable from a starting location so they could be marked as shared.

**Triggering Garbage Collection** Garbage collection is triggered using the \texttt{triggerGC} computation item:

\[
\text{op triggerGC : Nat} \rightarrow \text{ComputationItem}.
\]

This allows the language designer to decide how aggressive the collection policy should be. Currently, \texttt{triggerGC} has been added to the three equations in the memory operations that are used to allocate storage; no equations used to define KOOL language features have been modified. The \texttt{Nat} included in \texttt{triggerGC} contains the number of allocated locations. This is then used by the collector to decide when to begin collecting:

\[
\begin{align*}
\text{ceq } \text{threads}(t(\text{control}(k(\text{triggerGC}(N) \rightarrow K) \text{ CS}) \text{ TS}) \text{ KS}) \text{ ingc}(\text{GC}) \\
= \text{threads}(t(\text{control}(k(K) \text{ CS}) \text{ TS}) \text{ KS}) \text{ ingc}(\text{GC}) \\
\text{if } \text{runningGC}(\text{GC}).
\end{align*}
\]

\[
\begin{align*}
\text{ceq } \text{threads}(t(\text{control}(k(\text{triggerGC}(N) \rightarrow K) \text{ CS}) \text{ TS}) \text{ KS}) \\
\quad \text{ingc}(\text{noGC}(N')) \text{ gccount}(\text{GN}) \\
= \text{threads}(t(\text{control}(k(K) \text{ CS}) \text{ TS}) \text{ KS}) \\
\quad \text{ingc(gcClearMem) gccount(s(\text{GN}))} \\
\text{if } (N + N') \geq 1000.
\end{align*}
\]

\[
\begin{align*}
\text{ceq } \text{threads}(t(\text{control}(k(\text{triggerGC}(N) \rightarrow K) \text{ CS}) \text{ TS}) \text{ KS}) \\
\quad \text{ingc}(\text{noGC}(N')) \\
= \text{threads}(t(\text{control}(k(K) \text{ CS}) \text{ TS}) \text{ KS}) \text{ ingc}(\text{noGC}(N + N')) \\
\text{if } (N + N') < 1000.
\end{align*}
\]

The first equation just discards the trigger if the collector is already active. The second initiates collection when 1000 or more allocations have occurred\(^4\)—\(N\) being the number of new allocations, \(N'\) being the number already reported with prior \texttt{triggerGC}s. The last equation increments the number of reported allocations stored in \texttt{noGC} by the number of new allocations when the sum is less than 1000.

\(^4\)1000 was chosen after some experimentation, but further experimentation could show that a different number would be better. It may also be the case that there is no ideal number—hence the prevalence of collectors with generational policies, with each generation collected at different intervals.
<table>
<thead>
<tr>
<th>Test Case</th>
<th>GC Disabled</th>
<th>GC Enabled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time (sec)</td>
<td>Final Store Size</td>
</tr>
<tr>
<td>factorial</td>
<td>103.060</td>
<td>22193</td>
</tr>
<tr>
<td>ifactorial</td>
<td>97.100</td>
<td>21103</td>
</tr>
<tr>
<td>fibonacci</td>
<td>401.334</td>
<td>76915</td>
</tr>
<tr>
<td>addnums</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>garbage</td>
<td>259.500</td>
<td>32013</td>
</tr>
</tbody>
</table>

Single 3.40 GHz Pentium 4, 2 GB RAM, OpenSuSE 10.2, kernel 2.6.18.8-0.7-default, Maude 2.3. Times averaged over three runs of each test.

**Figure 6.5: KOOL: GC Performance**

**Evaluation** To evaluate the effectiveness of the garbage collector, five test cases were executed in KOOL, both with the collector enabled and disabled. Along with three numerical test cases, two test cases were added that were designed to generate a large amount of garbage. The test cases were:

- **factorial**, recursively calculating the factorial for 20, 40, ..., 180, 200;
- **ifactorial**, an iterative version of **factorial**;
- **fibonacci**, a recursive algorithm computing the fibonacci numbers from 1 to 15;
- **addnums**, which sums the numbers 1...100, 1...200, ..., 1...1000;
- **garbage**, which defines a class that holds an integer and then creates a temporary object of this class (which quickly becomes garbage) for the numbers 1...2000.

In all cases, the total execution time was recorded. Also recorded were the final size of the store and (in the cases where garbage collection was enabled) the number of collections that occurred. The performance results are shown in Figure 6.5.

At this point, results are mixed. The **factorial** and **ifactorial** tests do not appear to benefit from garbage collection – in both cases the collector slows performance down, even though it obviously shrinks the size of the store. The result for **fibonacci** shows little difference in execution time, although again the store is much smaller. In these three test cases, the cost of collecting either is higher than the benefit (**factorial, ifactorial**) or roughly equal to the benefit (**fibonacci**). However, in the final two test cases, **addnums** and **garbage**, garbage collection obviously helps. Without GC, **addnums** crashes; **garbage** completes in both, but is much faster with collection enabled.

One goal in developing the collector is to be able to reuse it in other languages. Based on the current design in KOOL, this should be straightforward. The only part of the collector that is language-specific is the operations and equations used to determine the set of root locations. The other parts of the collector definition are language-independent, and can be reused directly in any language that uses a similar (computation-based) definitional style.
6.2 Analysis Performance

The ability to model check and search programs using language definitions in rewriting logic is very closely tied to the performance of the definition. There are two general classes of performance improvement: improvements that impact execution speed, and improvements that impact analysis speed, which may even slightly reduce typical execution speed. Two examples of improvements are presented here, both of which have appeared in various forms in programming languages but not, to our knowledge, in other rewriting logic language specifications. First, auto-boxing is introduced to the language. This allows operations on scalar types, which are represented in KOOL as objects, to be performed directly on the underlying values for many operations (standard arithmetic operations, for instance), while still allowing method calls to be used on an object representation of the scalar where needed. Although mainly useful in dynamic languages like KOOL, this technique can also be used to perform automatic coercions between scalar and object types in statically-typed languages. Second, memory is segregated into two pools, a shared and an unshared pool. Rules are used when accessing or modifying memory in the shared pool, since these changes could lead to data races, while equations are used for equivalent operations on the unshared pool. This follows the intuition that changes to unshared memory locations in a thread cannot cause races. This change may or may not improve execution performance, but has a dramatic impact on analysis performance.

6.2.1 Auto-boxing

In KOOL, all values, including those typically represented as scalars in languages like Java, are objects. This means that a number like 5 is represented as an object, and an expression like 5 + 7 is represented as a method call. Primitive operations are defined which extract the primitive values "hidden" in the objects (i.e. the actual number 5, versus the object that represents it), perform the operation on these primitive values, and create a new object representing the result. This provides a "pure" object-oriented model, but requires additional overhead, including additional accesses to memory to retrieve the primitive values and create the new object for the result. Since memory accesses are modeled as rules in the definition, this also increases model checking and search time by increasing the number of states that need to be checked.

To improve performance, auto-boxing can be added to KOOL. This allows values such as 5 to be represented as scalars – i.e. directly as the primitive values. A number of operations can then be performed directly on the primitive representation, without having to go through the additional steps described above. For numbers, this includes arithmetic and logical operations, which are some of the most common operations applied to these values. Operations which cannot be performed directly can still be treated as message sends; the scalar value is automatically converted to an object representing the same value,
Figure 6.6: Example Definition Changes, Auto-boxing

which can then act as a message target to handle the method. Since boxing can
occur automatically, by default values, including those generated as the result of
primitive operations, are left un-boxed, in scalar form. This all happens behind
the scenes, allowing KOOL programs to remain unchanged.

An example of the rule changes to enable auto-boxing is found in Figure 6.6. The first equation is without auto-boxing. Here, when a floating point number \( F \)
is encountered, a new floating point object of class \texttt{Float} is created to represent
\( F \) using \texttt{newPrimFloat}. Any operations on this object, such as adding two floats,
will involve a message send. The next three rules are with auto-boxing enabled.
In the second equation, instead of creating a new object for \( F \), we return a scalar
value. The third equation shows an example of an intercepted method call.
When a method is called, the target and all arguments are evaluated, with the
method name held in the \texttt{toInvoke} computation item. Here, \( + \) has been invoked
with a target and argument that both evaluate to scalar float values, so we will
use the built-in float \( + \) operation instead of requiring a method call. In the fourth
equation, the boxing step is shown – here, a method outside of those handled
directly on scalars has been called with the floating-point scalar value as the
target, in which case a new object will be created just like in the first equation
([\texttt{wise}] will ensure that we will try this as a last resort). Once created, the
new object, and the values being sent as arguments (held in \texttt{boxWList}), will be
used to perform a standard method call.

Auto-boxing has a significant impact on performance. Figure 6.8 shows the
updated figures for verification times with this change in place. Not only is
this faster than the solution without auto-boxing in all cases, but it is now also
possible to verify deadlock freedom for up to 5 philosophers, which was not
possible with the prior definition.

6.2.2 Memory Pools

Memory in the KOOL definition is represented using a single global store for
an entire program. This is fairly efficient for normal execution, but for model
checking and search this can be more expensive than needed. This is because all
interactions with the store must use rules, since multiple threads could compete
to access the same memory location at the same time. However, many memory
accesses don’t compete – for instance, when a new thread is started by spawning
a method call, the method’s instance variables are only seen by this new thread,
not by the thread that spawned it. What is needed, then, is a modification to the

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definition that will allow rules to be used where they are needed – for memory accesses that could compete – while allowing equations to be used for the rest.

To do this, memory in KOOL can be split into two pools: a shared memory pool, containing all memory accessible by more than one thread at some point during execution, and a non-shared memory pool, containing memory that is known to be accessed by at most one thread. To add this to the definition, an additional global state component is added to represent the shared memory pool, and the appropriate rules are modified to perform memory operations against the proper memory pool\(^5\) Correctly moving memory locations between the pools does require care, however, since accidentally leaving memory in the non-shared pool could cause errors during verification.

The strategy we take to move locations to the shared pool is a conservative one: any memory location that *could* be accessed by more than one thread, regardless of whether this *actually* happens during execution, will be moved into the shared pool. There are two scenarios to consider. In the first, the spawn statement executes a message send. In this scenario, locations accessible through the message target (an object), as well as locations accessible through the actual parameters of the call, are all moved into the shared pool. Note that accessible here is transitive – an object passed as a parameter may contain references to other objects, all of which could be reached through the containing object. In many cases this will be more conservative than necessary; however, there are many situations, such as multiple spawns of message sends on the same object, and spawns of message sends on `self`, where this will be needed. The second scenario is where the spawn statement is used to spawn a new thread containing an arbitrary expression. Here, all locations accessible in the current environment need to be moved to the shared pool, including those for instance variables and those accessible through `self`. This covers all cases, including those with message sends embedded in larger expressions (since the target is in scope, either directly or through another object reference, it will be moved to the shared pool).

This strategy leads to a specific style of programming that should improve verification performance: message sends, not arbitrary expressions, should be spawned, and needed information should be passed in the spawn statement to the target, instead of set through setters or in the constructor. This is because the object-level member variables will be shared, while instance variables and formal parameters will not. This brings up a subtle but important distinction – the objects referenced by the formal parameters will be shared, but not the parameters themselves, which are local to the method, meaning that no verification performance penalty is paid until the code needs to "look inside" the referenced objects. Looking inside does not include retrieving a referenced object for use in

\(^5\)Note that, although this work follows the work on execution performance in this Chapter, it actually was performed at an earlier point in time, and thus uses an older model, with separate pools, instead of a model with a shared "bit" set on the memory location, as is discussed in the work on garbage collection in Section 6.1.
Figure 6.7: Example Definition Changes, Memory Pools

a lock acquisition statement (however, acquisition itself is a rule).

Figure 6.7 shows one of the two rules changed to support the memory pools (the other, for assignment, is similar), as well as part of the location reassignment logic. The first rule, which is the original lookup rule, retrieves a value \( V \) from a location \( L \) in memory \( \text{Mem} \). The location must exist, which accounts for the condition – if \( L \) does not exist, looking up the current value with \( \text{Mem}[L] \) will return \text{undefined}. \( \text{CS} \) and \( \text{TS} \) match the rest of the control and thread states, respectively. The second and third equation and rule replace this first to support the shared and unshared memory pools. The second is now an equation, since the memory under consideration is not shared. The third is a rule, since the memory is shared. This shared pool is represented with a new part of the state, \( \text{smem} \). The last three equations represent the reassignment of memory locations from the unshared to the shared pool, triggered on thread creation and assignment to shared memory locations. In the first, the location \( L \) and its value are in the unshared pool, and are moved to the shared pool. If the value is an object, all locations it holds references to are also added to the list of locations that must be processed. The second represents the case where the location is already in the shared pool. In this case, nothing is done with the location. The third equation applies only when all locations have been processed, indicating

<table>
<thead>
<tr>
<th>Ph</th>
<th>No Optimizations</th>
<th>Auto-boxing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>States</td>
<td>Counter</td>
</tr>
<tr>
<td>2</td>
<td>0.645</td>
<td>NA</td>
</tr>
<tr>
<td>3</td>
<td>0.723</td>
<td>NA</td>
</tr>
<tr>
<td>4</td>
<td>1.132</td>
<td>NA</td>
</tr>
<tr>
<td>5</td>
<td>6.036</td>
<td>NA</td>
</tr>
<tr>
<td>6</td>
<td>68.342</td>
<td>NA</td>
</tr>
<tr>
<td>7</td>
<td>895.366</td>
<td>NA</td>
</tr>
</tbody>
</table>

Single 3.40 GHz Pentium 4, 2 GB RAM, OpenSuSE 10.1, kernel 2.6.16.27-0.6-smp, Maude 2.2.

Times in seconds, Ph is philosopher count, Counter is time to generate counter-example, DeadFree is time to verify the program is deadlock free, state count based on Maude search results, NA means the process either crashed or was abandoned after consuming most system memory.

Figure 6.8: Dining Philosophers Verification Times
we should continue with the computation (with $K$).

This strategy could be improved with additional bookkeeping. For instance, no information on which threads share which locations is currently tracked. Tracking this information could potentially allow a finer-grained sharing mechanism, and could also allow memory to be un-shared when threads terminate. However, even with the current strategy, we still see some significant improvements in verification performance. These can be seen in Figure 6.9. Note that, in every case, adding the shared pool increases performance, in many cases dramatically. It also allows additional verification – checking for a counterexample works for 8 philosophers, and verifying deadlock freedom in the fixed solution can be done for up to 7 philosophers.

<table>
<thead>
<tr>
<th>Ph</th>
<th>States</th>
<th>Counter</th>
<th>DeadFree</th>
<th>States</th>
<th>Counter</th>
<th>DeadFree</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>35</td>
<td>0.64</td>
<td>0.798</td>
<td>7</td>
<td>0.621</td>
<td>0.670</td>
</tr>
<tr>
<td>3</td>
<td>244</td>
<td>0.694</td>
<td>3.610</td>
<td>39</td>
<td>0.637</td>
<td>1.287</td>
</tr>
<tr>
<td>4</td>
<td>1857</td>
<td>1.074</td>
<td>40.279</td>
<td>137</td>
<td>0.782</td>
<td>5.659</td>
</tr>
<tr>
<td>5</td>
<td>14378</td>
<td>4.975</td>
<td>501.749</td>
<td>634</td>
<td>1.629</td>
<td>34.415</td>
</tr>
<tr>
<td>6</td>
<td>111679</td>
<td>49.076</td>
<td>NA</td>
<td>2943</td>
<td>7.395</td>
<td>218.837</td>
</tr>
<tr>
<td>7</td>
<td>867888</td>
<td>555.791</td>
<td>NA</td>
<td>13670</td>
<td>47.428</td>
<td>1478.747</td>
</tr>
<tr>
<td>8</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>63508</td>
<td>325.151</td>
<td>NA</td>
</tr>
</tbody>
</table>

Figure 6.9: Dining Philosophers Verification Times (2)
Chapter 7

Policy Frameworks

Programs compute by manipulating *explicit* data, like integers, objects, functions, or strings, a process captured by dynamic semantics techniques like those shown in Chapters 3 and 4. However, program data may also have *implicit* properties which cannot be represented in the underlying programming language. For example, most languages have no way to indicate if a variable has been initialized, or if a pointer is never null. Some languages leave types implicit, providing no syntax to indicate types of variables or parameters.

Domain-specific examples are also common. A compelling example, commonly used in scientific computing applications, is units of measurement, where program values and variables are assumed to have specific units at specific points in the program or along specific execution paths. An example from security is information flow, where program data may have implicit security levels.

These implicit properties of program data give rise to implicit *policies*, or rules about how this information can be manipulated. For instance, one may require that variables be initialized on all paths before being read, or that only non-null pointers can be assigned to other non-null pointers. Languages with no explicit types generally still place type restrictions on operations such as arithmetic, where only values representing numbers can be used. Programs that use units of measurement must adhere to a number of rules, such as requiring two operands in an addition or comparison operation to have the same unit, or treating the result of a multiplication operation as having a unit equal to the product of the units of the operands (e.g., given *meter* and *second*, the resulting unit would be *meter second*). Applications concerned with information flow need to ensure that computations do not violate security requirements.

Because these properties are hard to check by hand, a number of techniques have been developed to make such implicit properties explicit and thus checkable using program analysis techniques. For instance, attached types have been introduced in Eiffel to indicate when references can be null, while various static analysis and type inference techniques have been applied to dynamically typed languages such as Self to discover type information needed for optimization.

Here, we focus on another common technique, the use of program *annotations*, either given by “decorating” program constructs (types, variables, function
names, etc.) with type-like information (type annotations); or by including additional information in special language constructs (function preconditions, assert statements, etc.) or special comments in the source code (code annotations). Many systems that use annotations are designed with specific analysis domains in mind; those that are more general often support either type or code annotations, but not both, or provide limited capabilities to adapt to new domains. This chapter presents a semantics-driven solution designed to overcome these limitations, policy frameworks. A policy framework is built around a language front-end and a core abstract rewriting logic or K semantics. The language semantics can be purpose built for analysis, but is often built by modifying an existing static or dynamic semantics of the language, abstracting concepts like value and modifying semantic rules so they work appropriately for static analysis (for instance, if the existing semantics defines standard execution behavior, the semantics used for analysis would be modified to take both branches of a conditional, tracking information along each).

Both the language front-end and the created core semantics are policy-generic, treating type and code annotations as black boxes. Individual analysis domains, such as units of measurement, then provide type and code annotations for the domain, as well as giving an algebraic definition of the domain data. These domains are both policy and, quite often, language generic, allowing them to be reused across different policy frameworks. Finally, individual policies are created as extensions to the policy framework, reusing inherited functionality, importing specific analysis domains, extending the provided annotation systems, and providing policy-specific semantics for language and annotation features.

Section 7.1 introduces analysis domains, providing examples both of domains specific to a given programming language (like type systems) and more general domains usable across multiple languages (like units of measurement). Section 7.2 then describes a policy frameworks for SILF, an imperative language also

![Figure 7.1: Multiple Polices in One Framework](image-url)
used in Chapter 6. In Section 7.2, two policies for SILF are presented, one for types (since SILF has no explicit types) and one for units of measurement. This illustrates reuse within a single policy framework, with multiple policies reusing the core framework functionality and inheriting multiple domains of analysis, a concept illustrated graphically in Figure 7.1 (as mentioned earlier, the policy framework core semantics may be based on an actual static or dynamic semantics for the language, but this is optional, and there is currently no way to automatically generate the policy semantics from an existing semantics). This chapter presents only some of the highlights of the policy; the complete specification for the SILF Policy Framework is provided online [80].

Looking forward, Chapter 8 will describe the C Policy Framework, or CPF, illustrating that the policy frameworks concept can be applied to real, widely-used languages. CPF will also support a policy for units of measurement using the same analysis domain used for SILF. This illustrates the reuse of analysis domains across policy frameworks, a concept illustrated graphically in Figure 7.2 (the same note applies here as applied in Figure 7.1).

### 7.1 Abstract Analysis Domains

Each analysis is based around an abstract analysis domain that indicates the values manipulated by the analysis. One domain can be used across multiple analysis policies and also across multiple policy frameworks. This section presents two analysis domains. The first, for types, is specific to the SILF language, providing a standard statically-checkable type system. The second, for units, is reusable across languages, providing a domain whose values are units of measurement. This domain is reused in Chapter 8 as part of the C Policy Framework UNITS policy.
The core policy support, shared across all languages with a policy framework, includes two sorts: PolicyVal and PolicyExp. These sorts represent generic policy values – the abstract values manipulated by a policy – and policy expressions, which are the logical formulae given in code annotations. Each policy is expected to define the actual values and expressions usable in the policy by adding operators which target these sorts.

Figure 7.3 shows the domain of types used in the SILF type checking policy, defined in Maude. Two modules are imported: TYPE-ANNOTATION-HELPERS, which provides operators for working with type annotations; and SILF-HELPING-OPS, which provides operators for working with the SILF configuration. Both modules map PolicyVal and PolicyExp to sort names more appropriate for the policy: PolicyExp to TypeExp, PolicyVal to Type.

Using these imported sorts, several additional operators are defined. SILF provides both integers and booleans. A new sort, BaseType, is created with two constructors, $int and $bool, representing these basic types. BaseType is a subsort of Type, allowing terms of sort BaseType to also be used as types (i.e., as policy values). The type $notype is given to represent situations where no type is given in a SILF program (for instance, on an unannotated variable declaration) – this is needed because types are not required in the language syntax. Finally, since SILF support arrays, a type for arrays, $array, is also defined. $array takes a base type as a parameter, meaning it is possible to form arrays of integers or booleans, but not of $notype or other array types.

7.1.2 Units of Measurement

Figure 7.4 shows the basic axiomatization of the units domain, used in the units of measurement checking policies for both SILF and C. Like with the types domain, shown above, the units domain renames sorts PolicyExp and PolicyVal, given them names more appropriate to the domain: UnitExp and Unit. A new sort, BaseUnit, is also introduced and made a subsort of Unit.
module UNITS is
  protecting RAT .
  including TYPE-ANNOTATION *
    ( sort PolicyExp to UnitExp, sort PolicyVal to Unit ) .

  sorts BaseUnit .
  subsort BaseUnit < Unit .

  ops $noUnit $fail $cons : -> BaseUnit .
  op "^" : BaseUnit Rat -> BaseUnit [prec 10] .
  op "*" : BaseUnit BaseUnit -> BaseUnit [assoc comm prec 15] .
  op "-" : Unit Rat -> Unit [ditto] .
  op "*" : Unit Unit -> Unit [ditto] .
  op NUnit : Nat -> BaseUnit .

  vars U U' : Unit . vars N M : Rat .

  eq U $noUnit = U .
  eq U $fail = $fail .
  eq U $cons = U .
  eq $fail ^ N = $fail .
  eq $noUnit ^ N = $noUnit .
  eq $cons ^ N = $cons .
  eq U ^ 0 = $noUnit .
  eq U ^ 1 = U .
  eq U U = U ^ 2 .
  eq U (U ^ N) = U ^ (N + 1) .
  eq (U ^ N) (U ^ M) = U ^ (N + M) .
  eq (U U') ^ N = (U ^ N) (U' ^ N) .
  eq (U ^ N) ^ M = U ^ (N * M) .
endfm

Figure 7.4: Units Domain

The rest of the module shown in Figure 7.4 defines a number of operators, used to construct units, and equations, used to determine when two units are equal.

Several "pseudo-units" are defined as base units: $noUnit, $fail, and $cons. $noUnit represents values with no assigned unit (for instance, loop counters iterating through an array); $fail represents a unit error, and is used as the result of an invalid computation; and $cons represents the unit of a constant, allowing constants to be used in standard calculations without explicitly declaring a unit. The next four operators define ways to build units: a unit (or base unit) can be raised to a rational power, and the product of two units, represented by placing the units together, is also a unit. Finally, the NUnit operator allows for the creation of unique "fresh" units, providing a way to generate units in situations where the actual unit is unknown, such as with unannotated function parameters. The use of globally unique units cuts down on spurious warnings, allowing operations like multiplication, which do not require the units to be the same, to be performed safely even on unknown units.

The equations then are used to normalize units and determine when two units are equal. For instance, $eq (U ^ N) (U ^ M) = U ^ (N + M)$ states that, given two units $U ^ N$ and $U ^ M$, the product of these units is the same as $U ^ (N + M)$.

Figure 7.4 then shows a number of actual units, built on the unit axiomatiza-
The SILF Policy Framework

The SILF language was introduced in Chapter [5] with the concrete syntax shown in Figure [6.1]. SILF provides a conceptually simple language in which to introduce the concept of policy framework, with enough complexities (global variables, function calls, arrays) to highlight challenges but without some of the difficulties of a language like C (function pointers, pointer arithmetic, aliasing, unions, etc).

7.2.1 Adding a Policy Framework to SILF

To create policies in SILF, the first step is to add the shared parts of a policy framework: an annotation-generic front-end and a core semantics.

SILF Policy Framework Front-end

The front-end for the SILF Policy Framework (SILF-PF) is an extended version of the existing SILF parser. Since SILF is a language that we have designed,
the language is extended directly to support policy frameworks, versus adding policy annotations through comments given in program source. The original syntax for SILF was given in Figure 6.1 in Chapter 6. Figure 7.6 shows just the modifications to this syntax made to enable policy frameworks.

Type annotations are identifiers with a leading $, like $int$ or $meter$, and are represented in the BNF with $tann$. $tvar$ represents a type variable, given with similar syntax: $$, instead of just $, like $$X. These are added in the syntax with Typed Identifiers, which also includes unannotated identifiers, and are then allowed in the productions for both variable and function declarations.

Code annotations are added by extending the language with new constructs for invariants on loops (for both $while$ and $for$ loops), assume and assert statements, and, in function declarations, function contracts with preconditions, postconditions, and modifies (which identifies the globals changed by a function). The syntax for each of these is given in the BNF: $PrePosts$ for function contracts (with short and long forms for preconditions, postconditions, and modifies), $Invariants$ for loop invariants, and extensions to the statement syntax for assumes and asserts. The code annotation, an arbitrary string, is itself represented with $ann$, and is actually parsed by the policy.

Each code annotation includes a policy tag, identifying the policy associated with the annotation language used in the annotation. This policy tag is just an identifier, given using $I$, and is given before the annotation, like in $pre(I)$.

**SILF Core Policy Semantics**

The core semantics includes the original SILF abstract syntax, extended with the new type and code annotation constructs mentioned above, and the configuration (i.e., K cells). The original dynamic semantics can be viewed as a special policy which ignores the additional constructs, with evaluation over a concrete, versus abstract, domain. Extensions for policies include modules providing: basic logical connectives; pretty printing capabilities for generating error messages; support for type annotation variables with limited forms of polymorphism; additional K cells for analysis information; and operators for working with these extensions.
K’s modularity allows new cells to be added without requiring changes to existing rules, making it easy to extend the state with new analysis information, such as line numbers (cell currLn), a copy of the environment current at function entry (cell old), and error messages generated by the analysis (cell log).

**Placeholders:** In addition to the extensions mentioned above, each SILF language feature is given a default generic semantics, which can then be extended as needed by individual policies. In some cases this semantics just provides support for part of a computation, expecting the policy to provide the other rules. For instance, addition expressions use the following generic rule:

\[
E + E' \rightarrow (E, E') \xrightarrow{\text{plus}} (E + E')
\]  

(7.1)

This rule says that, to process an expression like \(E + E'\), first evaluate \(E\) and \(E'\). To remember what to do with the results, the rule uses a new operator \(\text{plus}\), like what was described in Chapter 2. \(\text{plus}\) takes an expression, here the original expression \((E + E')\), which is kept solely in case it is needed to help generate an error message using a syntax pretty printer. In some other cases, the generic semantics provides specific hooks that are to be given meaning by policies. The rule for evaluating a number expression is:

\[
N \rightarrow \text{defaultIntVal}
\]  

(7.2)

Different policies can then give different definitions for \(\text{defaultIntVal}\), such as \$int\ for a types policy or \$cons\ (for constant) for a units policy. All analysis rules for handling standard SILF expressions (arithmetic, logical, and boolean operations) use similar techniques, deferring decisions about correctness to the actual policy.

**Statement Handling:** Statements in the policy framework for SILF are given a generic semantics, again often with hooks to policy-specific functionality. Figure 7.7 provides several examples of semantic rules for statements. Rule (7.3) is the generic rule for assignments. To check an assignment, the semantics
Figure 7.8: SILF Abstract Function Computations

first evaluates X and E. The computation item checkAssign then checks, in a policy-specific manner, whether the value of E can be assigned to X, based on the value of X. Rule (7.4) is similar, but also evaluates the array index expression, and then uses computation item checkArrayAssign to ensure the assignment meets all requirements for the policy. The final rule shown, Rule (7.5), shows the generic semantics for a conditional. The conditional guard, E, is evaluated first; the computation item checkIfGuard will then check the value in a policy-specific manner. Next, computations are built into the if computation item for both branches, ensuring that declarations and statements are both processed and that the environment is reset to the environment active before the branch to properly handle scoping.

Handling Functions: In the SILF dynamic semantics, a SILF program is processed by storing a computation for each function in a function table, fenv, and then invoking the main function’s computation (which can, in turn, invoke other functions in the table). In the policy semantics, all functions should be checked, which can be accomplished by invoking each in turn and evaluating each using the policy semantics. Function calls should not actually result in a transfer of control to another function. A second function table, ftenv, stores information about the function which can instead be used to check each call site. In summary, fenv and ftenv each have one entry per function; each entry in fenv is run once for each function f, to analyze the function; this analysis will trigger entries for each function f' in ftenv to be run once for each call to f' in f.

Figure 7.8 shows the abstract syntax for a function (in (7.6)) and the computations built for analysis that are stored as the function semantics in fenv and ftenv. The function consists of a potentially type annotated function name (TX), potentially type annotated function parameters (TXs), code annotations (PPl), and then the function declarations (Dl) and statements (Sl). The computations
are placed in the \( k \) cell as part of the analysis step, running the function for analysis, and referencing the parts of the function listed above (i.e., when a computation references TXs, this is the TXs from the function). The first two computations, the first present in \( fenv \) as the analysis semantics for the function, the second present in \( ftenv \) for analyzing individual call sites, are for policies without code annotations; the second two, involving the same cells, are for policies with code annotations.

In the first computation (7.7), \( checkRetType \) checks the return type annotations in a policy specific manner (the type checking policy defined below just verifies an annotation is given, for instance). \( checkSigTypes \) does the same with the formal parameters, while \( saveRetType \) retrieves the return type annotations and saves them in the state, allowing them to be checked at any return statements inside the function body. \( Dl \), a declaration list, is evaluated to process any local declarations in the function, while \( Sl \) is processed to evaluate the function body. At the end of function processing, \( dropRetType \) removes the return type annotations from the state.

The third computation (7.9) is very similar, with \( checkRetType \), \( checkSigTypes \), \( saveRetType \), and \( dropRetType \) all playing the same roles as in the first computation. \( applyPM \), given a list of code annotations \( PPl \), is used to apply the preconditions and modifies clauses for this policy. This occurs after type annotations are checked but before any declarations are processed in the function. Preconditions will assume facts given in their policy expressions, while modifies will unlock any listed global variables (which are all locked on function entry) so they can be changed inside the function. \( checkPoint \) will then capture the current state as the old state, so it can be referenced in other annotations. Next, \( savePostConds \) saves any postconditions in the state, similarly to how return type annotations are saved; this way they can be referenced during the processing of return statements, to ensure that, at function return, they hold. Finally, \( dropPostConds \) removes saved postcondition information when function analysis is done.

The second (7.8) and fourth (7.10) computations are much simpler. Both assume that a list of policy values, representing actual arguments to the function call, is provided. \( checkCall \) will then compare each provided value with the value of the formal parameter, looking for errors. Additional checking is also performed, such as verifying the correct number of arguments is given. \( checkAndBindCall \) is similar, but it will also bind policy values to the formal parameters, since they may be accessed in annotations. \( checkPreConds \) verifies that preconditions hold at call sites, while \( applyHavoc \) will clear any globals mentioned in a \texttt{modifies} clause, since the globals are assumed to hold a random value after the function call. Finally, \( getRetTypeForCall \) and \( getRetValForCall \) both retrieve the return value for the call, with the latter also taking any postconditions (with assignments to \texttt{@result}, representing the return value of the function) into account.
\[ i_1 + i_2 \rightarrow i, \text{ if } i \text{ is the sum of } i_1 \text{ and } i_2 \quad (7.11) \]

\[(\text{int}, \text{int}) \vdash +(E) \rightarrow \text{int} \quad (7.12)\]

\[(k) \quad (t, t') \vdash +(E) \quad \langle \text{.../k} \rangle, \quad \text{if } t /= \text{int} \text{ or } t' /= \text{int} \quad (7.13)\]

\[\text{issueWarning}(1, \text{msg}(E)) \vdash \text{int} \]

\[\text{if true then } Kt \text{ else } Kf \rightarrow Kt \quad (7.14)\]

\[\text{if false then } Kt \text{ else } Kf \rightarrow Kf \quad (7.15)\]

\[\text{bool} \vdash \text{if}(E, Kt, Kf) \rightarrow Kt \text{ \_ Kf} \quad (7.16)\]

Figure 7.9: SILF TCHECK Policy Rules

### 7.2.2 Defining A Type Checking Policy for SILF

Using the generic framework defined above, and the types domain defined in Section 7.1, the TCHECK policy adds types to SILF using type annotations, with policy checking then equating to type checking.

Rules for the policy semantics, in conjunction with the types domain, define the analysis. These rules generally following the dynamic semantics rules closely, with the addition that error checking logic has been added to catch errors that, dynamically, led to stuck states. For instance, Figure 7.9 shows the original integer addition rule for the SILF dynamic semantics, Rule (7.11), as well as two typing rules for addition. The first typing rule, Rule (7.12), indicates the expected scenario: each operand is of type \text{int}, with the entire operation also of type \text{int}. The second, Rule (7.13), is a type failure scenario; at least one of the types is not \text{int}, generating an error message (abstracted here as \text{msg}) of severity 1 (an error) using \text{issueWarning}. This rule also returns \text{int} as the type, versus returning a special failure type, to prevent a cascade of messages related to this one error (imagine this addition is part of a larger expression). Almost all expression rules follow this same pattern, with a success case and one or more failure cases generating appropriate error messages. For function calls, the rules are more involved, providing definitions of the operations given as part of the \text{ftenv} computation (for policies without annotations) shown in the core policy semantics above.

The fourth, fifth, and sixth rules show part of the semantics of conditionals in the original dynamic semantics and in TCHECK. Rules (7.14) and (7.15), part of the dynamic semantics, choose either the then (\(Kt\)) or else (\(Kf\)) branch, based on whether the condition is true or false. Rule (7.16), part of TCHECK, makes sure the condition evaluates to a boolean (\(E\), the original expression, in kept in case it is needed for error messages); after this check, the then branch and
the else branch are both checked. Rules for other statements are very similar, checking to ensure that arguments are of the correct type (for loops expect integers as the index start and end, for instance) and then checking the body of the statement.

Figure 7.10 shows an example run of the TCHECK policy. A program with type errors is shown in the top portion of the figure, with the generated error messages shown below. Policies define extensions to the default pretty printer operations to print their own type annotations, such as $int$ in the second and third error messages.

7.2.3 Defining a Units Policy for SILF

Again starting with the core semantics, the SILF-PF UNITS policy is designed to check for unit safety in SILF programs. A program is considered unit safe if it properly follows a number of unit rules, such as only adding values with matching units. These rules are codified in the UNITS policy semantics. The rules in UNITS use units as policy values, similarly to the way that TCHECK used types. The domain of units was presented in Section 7.1.

In many cases units can be treated the same as types, with similar semantic rules to those already shown for TCHECK. Figure 7.11 shows several UNITS rules. The first, Rule (7.17), is for addition, where, if both units match, the result is the same unit; Rule (7.18) is an error case for addition, where the units don’t match and the second unit isn’t a constant (which can be converted to any unit).

Rule (7.19) is a rule for multiplication, where the resulting unit is the product of the operand units. The fourth rule, Rule (7.20), is a rule for less-than, similar

```plaintext
1 function $int f($int x)
2   begin
3     return x + 1;
4   end
5 function $int main(void)
6   begin
7     var $int x;
8     x := 3;
9     x := f(x);
10    x := f(x,x);
11    if x then write 1; else write 2; fi
12    if (x < 5) then write 1; else write false; fi
13   end
```

Type checking found errors:
ERROR on line 10: Type failure: too many arguments provided in call to function f.
ERROR on line 11: Type failure: expression x should have type $bool$, but has type $int$.
ERROR on line 12: Type failure: write expression false has type $bool$, expected type $int$.

Figure 7.10: Type Checking using TCHECK
\[(u, u) \searrow + (E) \rightarrow u \quad (7.17)\]

\[\langle k \rangle (u, u') \searrow + (E) \ldots (/k), \text{ if } u =/= u' \text{ and } u' =/= \$\text{cons} \quad (7.18)\]

\[\text{issueWarning}(1, \text{msg}(E)) \searrow \$\text{fail} \]

\[(u, u') \searrow * (E) \rightarrow uu' \quad (7.19)\]

\[(u, u) \searrow < (E) \rightarrow \$\text{noUnit} \quad (7.20)\]

\[V \searrow \text{id}(E, Kt, Kf) \rightarrow Kt \searrow Kf \quad (7.21)\]

\[\langle k \rangle (u, u') \searrow \text{checkAssign}(X, E) \ldots (/k), \text{ if } u == u' \text{ or } u' == \$\text{cons} \quad (7.22)\]

\[\langle k \rangle (u, u') \searrow \text{checkAssign}(X, E) \ldots (/k), \text{ if } u =/= u' \text{ and } u' =/= \$\text{cons} \quad (7.23)\]

Figure 7.11: SILF UNITS Policy Rules

to addition but returning \$\text{noUnit}, since booleans do not have associated units (the error rule is similar to that for addition and is not shown).

Rules (7.21), (7.22), and (7.23) are rules for statements. Rule (7.21) handles conditionals, and is similar to Rule (7.16) in the TCHECK policy, except there is no need to check the value computed by the guard – any errors found in the guard expression will have already been reported, and the guard is not expected to have a specific unit (a more stringent requirement would be to enforce that the guard has no unit, but that is not done here). Rules (7.22) and (7.23) then show the regular and error cases for assignment. In Rule (7.22), the assignment is safe if the value being assigned either has the same unit or is a constant; in Rule (7.23), this condition does not hold, so an error message is issued.

An example run of a UNITS policy analysis is shown in Figure 7.12. Function id returns whatever it is given, making it polymorphic on the input unit. The errors on line 10 are standard arithmetic unit errors, while the error on line 13 is triggered by a conflict in the type of y and the postcondition of id.

Note that invariants have not come up in either policy. Up until now, they have not been needed. If the units assigned to a variable can change over time, for instance in each iteration of a loop, an invariant becomes essential. Since invariants are used in CPF, their discussion is deferred until Chapter 8.
function id(x)
  post(UNITS): @unit(@result) = @unit(x);
begin
  return x;
end
function main(void)
begin
  var $m x;
  var $f y;
  x := x + y;
  x := id(x);
  y := id(y);
  y := id(x);
end

Unit errors found:
ERROR on line 10: Assigning incompatible unit
  to explicitly annotated variable: x has
  unit $meter,(x + y) has unit $fail
ERROR on line 10: Unit failure, attempting to
  add incompatible units: (x + y),$meter,$feet
ERROR on line 13: Assigning incompatible unit
  to explicitly annotated variable: y has
  unit $feet, (id(x)) has unit $meter //

Figure 7.12: Unit Checking using UNITS
Chapter 8

The C Policy Framework

The C Policy Framework is an application of the Policy Frameworks concept, described in Chapter 7, to the C language. Section 8.1 describes the CPF frontend and annotation support, including a description of the process used to inline annotations in C code and generate function-level analysis tasks. Examples from the C abstract syntax used by CPF are then given in Section 8.2, while Section 8.3 describes the K cells used in the CPF analysis configuration. The policy-generic semantics are described in Section 8.4, with a special focus on the analysis semantics given C statements. Two policies are then presented. The first, the UNITS policy, is the CPF-equivalent of the UNITS policy for SILF shown in Chapter 7 and is shown in Section 8.5. Section 8.6 then shows a simple memory policy for guarding again null dereferences of C pointers. Finally, Section 8.7 presents a discussion of some of the challenges and limitations of the policy frameworks approach and the C Policy Framework.

8.1 CPF Frontend and Annotation Support

CPF supports both type and code annotations. Code annotations are provided as comments added to the original C source. CPF annotations are differentiated from standard comments with an @: /*@ for traditional “C-style” comments, //@ for line comments. Supported annotations include: precondition (pre), postcondition (post), and modifies (mod) in function headers; assume, assert, and invariant in function bodies; and tinvariant (type invariant) on structures and unions. Annotations also include the name of the policy, allowing different annotations to be added for different policies; annotations with policy ALL, or no policy, apply to all policies.

As a motivating example, Figure 8.1 shows an example of a simple function

```c
#include <stdlib.h>
int * f(int x) {
    int *p;
    p = (int*)malloc(sizeof(int));
    *p = x;
    return p;
}
```

Figure 8.1: A Potential Pointer Error
double sqrt(double x);

double lb2kg(double w) {
    return (10 * w / 22);
}

double e2s(double e, double w) {
    return sqrt(2 * e / w);
}

int main() {
    double pw = 5; double e = 2560;
    double speed = e2s(e, lb2kg(pw));
}

Figure 8.2: Implicit Units

that allocates a new integer pointer, assigns it the value of x, and then returns
the pointer. Although the intention is that this function should return a non-null
pointer, there is no way to indicate this. In fact the function violates this intent,
since malloc may return a null pointer (also causing a potential error on line 5).

Another example is provided in Figure 8.2. This time the example is a
fragment of a scientific application manipulating values with associated units of
measurement. Without any direct way to represent the units, even with a short
code fragment it is not clear that this code is unit safe.

Figure 8.3, an annotated version of Figure 8.1, shows an example of a CPF
type annotation. Type annotations can be added wherever types are used in the
program: on variables, function headers, structure definitions, casts, etc. Here, a
\texttt{\$notnull} annotation is added to the function type to indicate that the function
should always return a definitely not null pointer, a fact that can be used when
analyzing clients of this function.

Examples of CPF code annotations, including \texttt{pre}, \texttt{post}, and \texttt{assume}, are
shown in Figure 8.4, an annotated versions of the program fragment shown in
Figure 8.2. Type annotations are also used, since they can be mixed in the same
program. The figure shows examples of annotations (such as \texttt{\$kg} and \texttt{\$lb}) used
to identify the units associated with various program values.

Once the source code has been annotated, it is parsed to allow annotations
(in both styles) to be read and analysis tasks, one per function, to be generated.
Parsing takes place in two phases. First, a simple transformation is applied to
move annotations from comments or types into language syntax, producing a

#include <stdlib.h>
int * $notnull f(int x) {
    int *p;
    p = (int*)malloc(sizeof(int));
    *p = x;
    return p;
}
program using a CPF-extended C syntax that directly supports annotations. The second phase takes this modified version of the source, preprocesses it using a standard C preprocessor, then parses the preprocessed source using a modified version of the CIL parser for C \[145\], generating an internal, AST-like representation of the code. This internal representation knows about the CPF extensions, but does not know about the policy-specific annotation languages used in the annotations, allowing the parser to remain policy generic.

After parsing, two custom CIL passes prepare the code for policy analysis. The first simplifies function call sites and return statements by moving all computation before the call or return. The second replaces call sites with CPF statements to enforce the annotation semantics: assert for preconditions, assume for postconditions, and havoc, which assigns a random value, for modifies. Annotations on function headers are moved to the start of the function body, with preconditions becoming assumes and modifies becoming unlocks, which allows formal parameters and globals, locked against changes by default, to be modified. Postconditions become asserts at each return statement. Individual analysis tasks for each function are then generated in a format readable by Maude.

An example analysis task, for function e2s in Figure 8.4, is shown in Figure 8.5. It starts with 5 declarations, including for formal parameters and temporaries added by CIL; an empty unlock (because of an empty modifies clause); a checkpoint to save the starting state; and then the function body, including a call to sqrt (the location of the call is marked in case it is needed by the policy). sqrt has no precondition; it has an empty modifies, leading to an empty havoc, and its postcondition (see Figure 8.4) generates the assume. Finally,
an `assert` is inserted before the return, verifying that the unit of the value to be returned matches the annotation given for the postcondition. #CPFLine directives keep track of the current line, used in error messages.

Running CPF  The general process used to process analysis tasks with CPF is shown in Figure 8.6. First, an annotated source file is processed by the annotation processor and parser; this process generates multiple analysis tasks, one for each function defined in the source file. These analysis tasks, along with the policy to be checked, are then handed to Maude; the core and policy semantics are read first, with the analysis tasks, encoded as Maude terms, then each evaluated. When this process terminates, the results generated by policy checking are displayed to the user. Messages logged during analysis include a number representing the severity of the message (i.e., 1 for error, 2 for warning, etc.), and can be filtered appropriately, allowing a policy to define a severity for

---

1. `CPFIAssert`, used here, is like a normal `assert`, but is executed in the starting environment captured when the state is checkpointed. `crrnt` is then used to access the current, not the starting, environment. Another option would be to make `crrnt` the default and instead provide an operation like `old` to access the initial state.
8.2 Abstract Syntax

An extended abstract syntax of C is defined as part of the policy framework to allow semantic rules and equations (hence referred to as just rules unless the distinction is important) to be written over a syntax as close to native C syntax as possible. Syntactic categories, such as expressions and statements, are defined as sorts, with operations defined for each syntactic construct, generally using mixfix notation, which allows a style of definition accessible to those familiar with notations such as BNF. Syntactic constructs include abstract forms of standard C language constructs and constructs added by CIL during analysis task generation, including: new statements for assume and assert annotations; representations of abstract values, such as the unit values shown in Figures 7.4 and 7.5 and syntax for expressions used inside annotations, such as the @unit annotation shown in Figure 8.4. In addition, a number of “helper” operations written to work with the abstract syntax have been defined, to perform tasks such as checking whether an identifier represents a formal parameter or whether a declaration of a variable is of a structure type.

Figure 8.7 shows several examples of the CPF C abstract syntax with the extensions mentioned above. The first operation defines the + operator as taking two expressions (one for each underscore) and yielding an expression; the second and third similarly define the if (including an else clause) and return statements. The fourth is for the assume statement, added to programs during CIL processing to represent assume annotations. Finally, the fifth is the @unit operation, part of the UNITS policy, used inside annotations to get or set the unit associated with a C object (a memory region that can be read or written).

8.3 K Cells

CPF includes the definition of a generic computation used by the semantic rules to keep track of information needed by or produced during analysis. A number of individual K cells make up this computation, including: the abstract computation, k; the current environment, env; the environment on function entry, origenv, used in annotations generated for postconditions or where the
annotation requests the old value of an expression; a set of all environments, $\text{envs}$, the need for which is explained below; $\text{out}$, containing output messages generated during policy checking; $\text{nextLoc}$, holding the next (abstract) memory location to allocate; $\text{currLn}$, the current line number from the original source program; $\text{structMap}$, containing definitions of all structures and unions used by the function being analyzed; and $\text{gotoMap}$, used when processing $\text{goto}$ statements to keep track of jump destinations. Individual policies can extend this with information needed by the policy semantics. Rules will then change the information in one or more of these cells as analysis proceeds, such as by adding new mappings from names to values in $\text{env}$ or modifying the abstract computation in $k$. A graphical representation of the CPF configuration is shown in Figure 8.8.

### Environments

CPF statements and expressions are each evaluated in the context of an environment, the definition of which is shown in Figure 8.9. An individual entry in the environment contains an identifier (set to a default value for C objects that are not associated with a distinct identifier, such as $\text{list}\rightarrow\text{next}\rightarrow\text{x}$), an abstract location, an abstract value, and $\text{EnvItem}$, which is a set of additional environment information which can be extended to meet the needs of a specific policy. The concept of $\text{Value}$ is also policy-specific, and can represent individual values or collections of values (lists, sets, etc.). These individual entries are combined into a set to make up the program environment, with operators and structural equations provided to retrieve the value at a specific name or location.

CPF statements then each return a set of environments as a result of execution, with each environment in the set being the result of execution along a distinct control flow path (i.e., the branches of a conditional, the cases of a $\text{switch}$, paths created using $\text{goto}$, etc.). Merging the different environments in the set is then done in a policy-specific manner. For instance, some policies may want to combine returned values, for instance by using $\top$ and $\bot$ to represent over or under-determined values; others may want to track a set of possible values,
creating this set based on the values in the different returned environments; other policies may want to track the individual environments to track related changes to different objects, potentially improving the precision of the analysis at the cost of increased analysis time. CPF provides logic to evaluate statements with sets of environments, stored in \texttt{envs}, by evaluating each statement in each environment in the set, allowing a simplified per-statement semantics which assumes that a statement is evaluated in just one environment at a time.

### 8.4 Abstract Evaluation Semantics

CPF includes a collection of semantic modules for reuse in specific policies. The semantic modules provide: generic rules designed to manage the configuration or provide often-needed functionality, such as rules to issue warning messages; definitions of, and generic functionality for working with, expressions used in annotations; rules for breaking up expressions into subexpressions and (at least partially) evaluating them, with the rules used to combine evaluation results specific to a policy (for instance, CPF rules break a plus expression into the left and right operands, while rules in the UNITS policy determine if the resulting values can be added together); default definitions of many C abstract values, such as pointers and structures; and rules for C statements.

**Simple Statements** Some C statements, given one environment as input, just yield one environment in return. This includes expression statements, null statements (i.e., a lone \texttt{;}), return statements, break, and continue. In many policies, expression statements will be the source of most checks and potential errors, since they include most assignments and calculations. The semantic rule for expression statements is shown in Figure 8.10. To process each statement, CPF pairs it with each environment from the environment set. Given this pair, with statement \texttt{E ;} and environment \texttt{Env}, the rule evaluates expression \texttt{E} in environment \texttt{Env} (\texttt{Env} is made current by putting it into the \texttt{env} cell), the value it evaluates to is discarded as it is not needed by the analysis, and the current environment, containing any changes made by \texttt{E} to the abstract values assigned to C objects, is captured as the result of statement evaluation.

The \texttt{return, break, and continue} statements all cause abrupt changes in control flow, but do not create multiple environments. This change in control

```plaintext
1  op noEnv : -> Env .
2  op _-_ : Env Env --> Env [assoc comm id: noEnv] .
3  op [_,_,_,_] : Identifier Location Value EnvItem --> Env .
4  op [_,_] : Env Identifier --> Value .
5  op [_,_] : Env Location --> Value .
6  7  (Env [X_,_,V,_,_])[X] = V .
8  (Env [_,L,V,_,_])[L] = V .
```

Figure 8.9: CPF Environments
flow is modeled using an environment lock: the environment is locked against changes until it is unlocked by another semantic rule, such as (for break) the end of a switch or while statement, or (for return) the end of the function body. Figure 8.11 shows the semantics of break and return E, with the rules for continue and return (with no expression) being almost identical. In Rule (8.2), break causes the environment to be locked with label 'break using the lk operation. In Rule (8.3), return E; first evaluates expression E, with the resulting environment then captured and locked using the captureLockEnv computation item. The final rule, Rule (8.4), shows an example of environment unlocking: when a switch statement has no more cases, indicated using empty, any environments locked with break are then unlocked, since they would again be “active” after the switch, with this set of unlocked environments being the result of the switch computation.

Complex Statements Other C statements, given one environment as input, can generate multiple environments due to multiple branches and nested statements. This includes conditionals, loops, the switch statement, and the goto statement. Figure 8.12 shows the semantics of the conditional. In Rule (8.5), E is evaluated, while S and S', the branch bodies, are saved for later using the if computation item. When E has been evaluated, in Rule (8.6) the then branch of

Figure 8.12: CPF Conditional Semantics
the conditional is evaluated in the environment available after the evaluation of \( E \). This environment is also saved with the else branch in the \textbf{else} computation item, since the else body will also be evaluated in this environment. Note that the current environment is moved from the \textit{env} to the \textit{envs} cell; this is because the statement-handling logic always starts in an environment set, executing the statement in each environment. Once the then branch finishes the \textbf{else} computation item will be evaluated using Rule (8.7); the set of environments, \( ES \), generated by the then branch is saved using the \textit{mergeEnvs} computation item, which will take the set of environments \( ES \) and the set generated by the else branch and merge them as the overall result of executing the conditional.

The semantics of \textbf{switch} are similar, except there are an arbitrary number of branches. Each branch will begin evaluation in the environment available after the \textbf{switch} expression is evaluated, plus in any environments that “fall through” from an earlier branch. This takes advantage of the environment locking capability described above; environments locked on ‘break will not be used in later branches, but will then be unlocked at the end of the \textbf{switch}.

Figure 8.13 shows the rules for the semantics of \textbf{while} (CIL changes all loops to \textbf{while} loops). In Rule (8.8), the loop expression (E), loop body (S), and current set of environments (ES) are all saved using the \textit{repeat} computation item, to be iterated twice (once for any initialization done in the loop body, once to check convergence). Rule (8.9) then shows evaluation of the \textit{while[E,S]} statement, which evaluates \( E \) and then, in Rule (8.10), evaluates \( S \) in the resulting environment (this environment is also saved, since it would be the loop result in cases where \( E \) evaluates to 0 in an actual execution). The last three rules then
define how the loop is repeated. In Rule (8.11), the resulting set of environments is the same as the starting set, meaning the loop converged, so the program continues. In Rule (8.12), the environment sets differ; with at least one iteration left, the loop is repeated using the old and new sets of environments (| is the set formation operator), with any environments locked by a continue statement unlocked. In Rule (8.13), there are no iterations left and the environment set has not yet converged, so a warning is issued (with msg standing in for the actual warning message), indicating that the analysis may not be sound due to potential missing mappings to abstract values in the environment. This handles common cases, where the loop values stabilize quickly (for instance, where objects with one value are assigned another in the loop body, stabilizing by the second iteration), without needing an actual loop invariant, while still allowing programmers to add one using annotations in cases where it is needed.

The goto semantics make use of the gotoMap cell, a map from labels to a tuple of information, including the computation from the point of the label forwards, environments active at the time the label is jumped to or encountered during evaluation, and environments that have already been checked. To evaluate the goto, the set of environments active at each label are accumulated during evaluation of the function body. At the end of the function, each entry in the goto map, for each label, is checked to see if any active environments have not yet been checked. If so, the semantics will “jump” to that part of the code and evaluate it with the unchecked environments. Like the while loop, this will by default be done at the most twice, with the values either stabilizing or diverging. In the latter case, a similar warning is generated for the same reason. The rules for the goto semantics can be found with all CPF rules in the downloadable CPF semantics [79].

**CPF-Specific and Other C Statements** CPF adds several statements to the C language used during analysis. This includes assert, assume, invariant, and a number of directives used to provide additional information to the semantics, such as #CPFLine for line numbers (used in error messages) and #CPFWarn for warning messages generated by CIL, such as when CIL cannot determine which function is being called through a function pointer. The semantics of assert, assume, and invariant are at least partially left to each policy to define, since the expressions used in each are defined as part of the annotation language for the policy. Other C statements, not discussed above, include C statement sequences, where each statement is evaluated in turn, using the environment set resulting from evaluating the first statement as the input to evaluating the second statement; and blocks, which are evaluated by evaluating the body of the block. There is no need to model scope for blocks, since CIL moves all declarations to the start of the function body, renaming variables if needed to maintain scope.
8.5 The CPF UNITS Policy

The units of measurement policy, UNITS, is used to analyze programs for unit safety violations, i.e. for C expressions that use units in unsafe ways, such as adding two incompatible units. UNITS uses values from the abstract units domain, defined in Figures 7.4 and 7.5, and uses an annotation language based on the defaults provided with CPF, extended with an operator $\texttt{@unit}$, which sets (for assume) or calculates (for assert) the unit assigned to an expression. Type annotations allow all predefined units, including those with rational exponents, to be used, and also allow type variables to be used to indicate relationships among unknown units.

8.5.1 Unit Analysis

Analysis starts by generating one analysis task for each function, a process discussed above in Section 8.1. Once these tasks have been generated, each is checked using the CPF + UNITS semantics in Maude. Checking is accomplished using abstract execution over the units domain, with expressions manipulating values representing units, pointers, structures, etc. Statements are executed similarly to a concrete semantics, except that all paths in branching statements are taken, and looping statements are executed at most a fixed number of times. Assertions and assumptions written in the CPF + UNITS assertion language are also checked as the program is executed, using annotation language-specific semantic rules. An important point here is that units used in these annotations are not restricted just to ground terms; $\texttt{@unit(E)}$ can be used to get the unit of any C expression, and $\texttt{unit}$ expressions can be put together in complex ways, such as saying the unit of a variable, squared, is equal to the product of the units of two other variables.

Declarations

The CPF + UNITS rules for handling declarations provide an initial abstract representation of memory for the global variables, formal parameters, and local variables of a function. Different allocators are used for each C object type, properly structuring the memory for later use in abstract C statement and expression execution. For instance, the allocator for scalars associates a unit with the scalar, while the allocator for pointers associates a reference to a memory location with the pointer. Structures and unions are represented as maps from names to other locations, so a structure $s$ with field $x$ would be represented as a map from $x$ to the location holding the abstract value of $x$. Allocation is recursive; structure allocation also allocates structure fields, while allocation of arrays and pointers allocates the base type of the array or pointer as well. Currently, unions are represented like structures. One challenging but common case to represent is structures which contain pointers to other structures. It is
not possible to allocate the entire memory representation up front, since this
could be (in theory, at least) infinite. Pointers inside structures are instead
created with an allocation trigger, which will allocate the pointer’s target on
the first attempt to access it. This allows the memory representation to grow
sensibly, modeling just what is needed to perform the unit safety analysis.

Initial values are given to local variables using a combination of assertions
(from the annotations) and assignments. For instance, an assertion may indicate
that variable \( x \) has unit \( \text{meter} \); a declaration like \texttt{int y = x;} would then
associate \( \text{meter} \) with \( y \) as well. Initial units for formal parameters and global
variables are based just on the function preconditions. If a precondition or
assignment does not indicate the initial unit of a variable, it is assigned a globally
unique unit, which will ensure it is used correctly in expressions (described below).
Once initial values have been assigned to formal and global parameters, a locking
process locks certain memory locations to make sure they cannot be changed in
ways that are not reflected in the annotations. For instance, it is not possible to
write a new unit through a pointer given as a formal parameter. This ensures
that changes visible outside the function but not included in the preconditions
and postconditions are prevented, allowing checking to be truly modular. It is
possible to override this locking behavior using the \texttt{modifies} clause, which will
ensure the proper behavior (setting modified memory to unknown values) for
callers.

**Statements**

During abstract execution, different units can be assigned to the same C object
on different execution paths through the function body. For instance, the code
element in Figure 8.14 assigns initial units to \( x \) and \( y \) before a conditional
statement, and then assigns new units in the \texttt{true} branch of the conditional (not
changing the units if the conditional is not entered). To model this path-sensitive
aspect of unit assignment, a set of environments is used, with each environment
in the set representing one distinct assignment of units to C objects. Statements
in CPF + UNITS are then treated as “environment transformers” which, when
given an environment, can generate a new environment set as a result. Starting
with the first statement in the function body, each statement is executed once
for each environment in the set; each such execution yields a new environment
set, with the union of these environment sets then taken as the overall result of
executing the statement. Executing each statement in a single environment at a
time allows the semantics to be simpler, with a closer relationship to a concrete
semantics of C.

Several statements in C represent just straight-line code. Executing these in
CPF + UNITS with one input environment yields a set with just one output
environment. These include expression statements, break statements, continue
statements, return statements, and the assert and assume statements added
int x, y, z; //@ assume: unit(x) = unit(y) = m
if (b) {
    y = 3; //@ assume: unit(y) = f
    x = y;
}
z = x + y;

Figure 8.14: Path-Sensitive Unit Assignment

to model the assert and assume annotations. Expressions can never “split” the environment: any expressions that can do so (ternary operator, short-circuit evaluation) are transformed first in CIL into equivalent expressions using statements and temporaries. Other statements in C can cause environment splitting in the CPF + UNITS semantics, where the statement can possibly return a set of environments containing multiple distinct environments. These include conditionals, loops (CIL transforms all loops into while loops, so for and do-while loops do not need to be modeled), switch statements, and goto statements. In these cases, given a single input environment, an environment set of arbitrary size could be returned.

In the case of if and switch statements, the environments generated by each branch are merged together at the end of the statement. For an if, a set of environments can be generated for each branch, which could also contain conditionals, loops, etc. For a switch, each case can generate a set of environments; also, if there is a path through a case that does not break, the environments generated along that path fall-through to the following case.

Loops and gotos require more complex handling. For both, the body of the construct (in the case of a goto, the code after the target label) is analyzed twice: the first pass discovers changes that can occur when first evaluating the construct, while the second discovers changes that occur between iterations. If convergence is not achieved, a warning message is printed, indicating that there may be an error. In cases where units do not change, convergence occurs immediately, and no error is reported. Loops repeat just the loop body and the loop test. Gotos take advantage of the semantics approach used in the CPF, where computations can be saved and restored. In standard language definitions, this feature can be used to model language features such as continuations or exceptions; here, it is used to save the computation from the point of the label, which can then be replayed, with different environments, repeatedly.

Again looking at Figure 8.14, the use of environment sets adds some additional expressive power. Here, since x and y are both changed in the same way on the same path, when they are added and assigned to z CPF + UNITS can determine that both either have unit m or unit f. If, instead of environment sets, sets of units were assigned to x and y, both could be either m or f, and combinations where x is one and y the other would yield a false positive. Unfortunately, there is also a cost to this feature – certain pathological situations can cause the
environment set to expand exponentially, such as programs with deeply nested branches of conditionals that modify units differently along each branch. This rarely occurs, though, since while values change often in the concrete domain they change rarely in the abstract domain – it is more common to assign a new number to a variable than a new unit. Also, while our approach provides greater flexibility and generality, it is possible to maintain a discipline that either treats units as types, disallowing changes once a unit is assigned, or that otherwise considers such cases to be unit failures, ensuring that at most one environment is created and avoiding problems with environment set expansion (although this can come at the cost of generating more false positives). In cases where the set does grow, our approach is to set a high-water mark on the size of the environment set; any time the size of the set goes beyond this mark, enough environments are discarded to stay under it and an error message is generated, indicating that the analysis results are unsound.

**Loop Invariants**

One way to attempt to force loop stabilization is to use a loop invariant. In CPF, invariants work similarly to other systems with invariants: an invariant is checked at the start of the loop to ensure it holds, and is then checked at the end of the loop to ensure it is reestablished along all branches inside the loop. In the UNITS policy, the invariant also assigns fresh units to any C objects mentioned in the invariants. These are assigned after the loop exists, with assignment done in such a way that equalities dictated by the invariant continue to hold.

Figure 8.15 provides a simple example of a loop with an invariant. Without the invariant, the loop would fail to stabilize, as the units assigned to both \( x \) and \( y \) change during each iteration. Given that both start with the same unit, the invariant does hold: the unit of \( x \) is equal to the unit of \( y \) before the loop starts and after each iteration. Once the loop completes, a new fresh unit is assigned to \( y \), since the final unit of \( y \) is unknown. The model the fact that the invariant held at the end of the loop, the same unit is assigned to \( x \). Because of this, the assignment to \( y \) on line 10 is correct, even if the specific unit is unknown.

In the example in Figure 8.16, while the invariant is correct, it needs to
int main(int argc, char* argv[]) {
    int x; //@ assume(UNITS): @unit(x) = $m
    int y; //@ assume(UNITS): @unit(y) = $m
    int z; //@ assume(UNITS): @unit(z) = $m
    int n = 0;
    //@ invariant(UNITS): @unit(x) = @unit(y)
    while (n < 10) {
        x *= x;
        y *= y;
        z *= z;
    }
    y = x + y;
    return 1;
}

Figure 8.16: Loop Invariants, Incomplete Invariant

be strengthened to include z in order to allow the loop to stabilize, since z is changed at each iteration. As is, an error is reported, indicating that the loop does not stabilize and the remaining analysis may be unsound.

Expressions

To evaluate expressions in CPF + UNITS the semantic rules need to properly modify, combine, and propagate abstract values representing units and C objects (pointers, structures, etc). Expressions, along with assert statements, are also the point where unit safety violations are discovered, so semantics for expressions which can produce failures need to ensure that the failures are properly handled. Figure 8.17 includes rules for a representative set of expressions, illustrating how abstract values are propagated and failures are detected.

Rule (8.14) models the multiplication operation. Here, given two unit values U and U', the result is their product, U U'. Rules (8.15) and (8.16), for addition, are structured similarly, but must also check that the combination of the units is correct. This is done using compatible: if one unit is $cons, or both units match, they are compatible, but otherwise they are not. Rule (8.15) represents the case where the units are compatible and thus can be added, with the merge operation deciding on the new unit to be returned (merge ensures that, if one unit is $cons, that unit is not returned as the resulting value). Rule (8.16) represents the case where the units are incompatible, which will generate an error message and yield $fail are the resulting unit. Rule (8.17) shows how the greater-than relational operation is handled, and is similar to the rules for addition: to compare two values they must have the same unit. However, the returned unit is $noUnit, since it doesn’t make sense to assign a unit to the result of the comparison.

Rule (8.18) is used for assignment. The value evaluates to an 1vp, or location-value pair, with the location and current value of the object being assigned into. The value of the right-hand expression is assigned over the current value to the same location using assign. While this works for units, it also works for other C entities, such as the representations for pointers and structures. Rule (8.19) then shows the semantics of +=, which is a combination of the rules for + and
\[ (k) \text{merge}(U, U') \leadsto \text{assign}(L) \]

(8.14)

\[ (k) U + U' \leadsto \text{merge}(U, U') \text{ if } \text{compatible}(U, U') \]

(8.15)

\[ (k) \text{issueWarning}(1, \text{msg}(+)) \Rightarrow \text{fail} \text{ if } \text{compatible}(U, U') \]

(8.16)

\[ (k) U > U' \leadsto \text{noUnit} \text{ if } \text{compatible}(U, U') \]

(8.17)

\[ (k) \text{lvp}(L, V) = V' \Rightarrow \text{assign}(L) \]

(8.18)

\[ (k) \text{lvp}(L, U) + = U' \Rightarrow \text{assign}(L) \text{ if } \text{compatible}(U, U') \]

(8.19)

\[ (k) \text{struct}(X, (\text{sfield}(X, L'))).X \Rightarrow \text{lookup}(L') \]

(8.20)

\[ (k) \text{struct}(X', (\text{sfield}(X, L')).X \Rightarrow \text{lookup}(L') \]

(8.21)

Figure 8.17: Units Expression Rules, in C

8.5.2 Minimizing Annotations

Techniques similar to those used in the earlier work on C-UNITS [156] are used to minimize the number of annotations needed in a program. This includes the use of default “fresh” unit values, which provide default, incompatible units in cases where no specific unit is enforced (addition of two fresh units would fail, for instance); support for loops that stabilize quickly without the requirement to add loop invariants; a rich annotation language which can be used to make complex

assignment, performing both the compatibility check and the assignment to the location of the lvalue. In this case, the values should be units, since it is necessary to compare them to verify the operation is safe (in general, for many of these rules alternate rules are needed for pointers, to take account of pointer arithmetic).

Finally, rules [8.20] and [8.21] show how some aspects of pointers and structures are handled. A pointer is represented as a location – a pointer to location \( L \) has the value \( \text{ptr}(L) \). On dereference, the location held in the pointer is looked up to retrieve its value. A structure is represented as a tuple containing the name of the structure type, a set of structure fields, and additional declaration information not used in the rule. When field \( X \) is looked up in a structure using dot notation, the location of \( X \) is retrieved and then looked up to bring back the value.
assumptions involving multiple C objects; the ability to use constants with values of any unit; and the flow of units from values on assignment, allowing the reuse of temporaries without the need to annotate the change. Other techniques, involving improvements in the handling of common loop scenarios, are the subject of future research.

8.5.3 Evaluation

Evaluation was performed using two sets of experiments. All tests were performed on the same computer, a Pentium 4 3.40 GHz with 2 GB RAM running Gentoo Linux and Maude 2.3. In the first, the focus was on performance, ensuring that using a per-function analysis would scale well as desired. The results are shown in Figure 8.18. Here, each test performs a series of numerical calculations: straight includes straight-line code; ann includes the same code as straight with a number of added unit annotations; nosplit includes a number of nested conditionals that change units on variables uniformly, leaving just one environment; and split includes nested conditionals that change variable units non-uniformly in different branches, yielding eight different environments in which statements will need to be evaluated. LOC gives the lines of code count, derived using the CCCC tool [110], for each function, with the same function repeated 100, 400, or 4000 times.

As shown in Figure 8.18, performance scales almost linearly: quadrupling the number of functions to check roughly quadruples the total processing time. Per-function processing time is small, making CPF + UNITS a realistic option for checking individual functions during development, something not possible in some other solutions (such as C-UNITS) that require the entire program be checked at once. Splitting environments increases the execution time, but not prohibitively: with eight environments, the time per function to process split is roughly double, not eight times, that to process nosplit, which has just one environment. Finally, processing annotations in the units annotation language seems to add little overhead; annotations are treated as statements during processing, so in some sense just add additional “hidden” lines of code.

The second set of experiments compares against some of the same exam-


<table>
<thead>
<tr>
<th>Test</th>
<th>Prep Time</th>
<th>Check Time</th>
<th>LOC</th>
<th>Functions</th>
<th>Annotations</th>
<th>Errors</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex18.c</td>
<td>0.113</td>
<td>0.764</td>
<td>18</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>fe.c</td>
<td>0.113</td>
<td>0.796</td>
<td>19</td>
<td>2(3)</td>
<td></td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>coil.c</td>
<td>0.113</td>
<td>59.870</td>
<td>299</td>
<td>4(3)</td>
<td></td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>projectile.c</td>
<td>0.122</td>
<td>0.882</td>
<td>31</td>
<td>5(2)</td>
<td></td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>projectile-bad.c</td>
<td>0.122</td>
<td>0.866</td>
<td>31</td>
<td>5(2)</td>
<td></td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>big0.c</td>
<td>0.273</td>
<td>2.223</td>
<td>0</td>
<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>big1.c</td>
<td>0.998</td>
<td>22.853</td>
<td>0</td>
<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>big2.c</td>
<td>33.144</td>
<td>381.367</td>
<td>96611</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Times in seconds. All times averaged over three runs of each test. Function count includes annotated prototypes in parentheses. FP represents False Positives.

Figure 8.19: CPF + UNITS Unit Error Detection

Examples used by Osprey [93] (a unit checker discussed in more detail in Chapter 9), some of which were originally from C-UNITS, with the results shown in Figure 8.19. fe.c is an energy calculation; coil.c is part of an electrical inductance calculator; projectile.c calculates the launch angle for a projectile; and projectile-bad.c does the same, but with an intentionally-introduced unit error. big0.c, big1.c, and big2.c include a repeated series of arithmetic operations and are designed to test the size of function that CPF + UNITS can handle, with big2.c included as an especially unrealistic example.

Overall, Figure 8.19 shows that the annotation burden is not heavy: assumptions on variable declarations are sometimes needed, while preconditions and postconditions are often needed, with the number of annotations needed by Osprey being similar (although those used by Osprey are generally smaller). big0.c, big1.c, and big2.c require no annotations, while coil.c requires 14, including on function prototypes. fe.c requires 9 annotations, with ex18.c requiring 10. The projectile.c example is particularly interesting: the use of a more flexible annotation language allows a more general version of the program to be checked than in Osprey (which cannot relate parameter and return value units), maintaining unit polymorphism, while projectile-bad.c includes an error not caught by Osprey, since the error involves using a variable with a different unit (pounds versus kilograms) in the same dimension. Overall, only 16 annotations are needed across 5 functions and 2 prototypes in both projectile.c and projectile-bad.c. coil.c shows a disadvantage of the CPF + UNITS approach: one of the goto statements never stabilizes, meaning the units keep changing with each iteration. This raises an error in the program, which in this case appears to be a false positive.

8.6 Case Study: Null Pointer Analysis

The use of explicit memory management in C provides for improved performance, but at the cost of several common errors, including memory leaks, multiple frees, and dangling pointers. This section presents a policy for dealing with another, quite common potential problem, attempts to dereference pointers which may point to null. This policy, NOTNULL, includes an annotation language for
Value  
\[ V ::= \$\text{undefined} \mid \$\text{defined} \]  

Value Type  
\[ VT ::= \$\text{zero} \mid \$\text{other} \mid \text{ptr} L \]  

Location  
\[ L ::= \$\text{null} \mid \$\text{notnull} N \]  

Figure 8.20: Not Null Policy: Policy Domain

specifying the “nullness” of pointers passed as function parameters and returned as function results. The policy presented here is part of a larger policy, inspired by the LCLint tool [46], that is currently under development.

The NOTNULL policy uses the default abstract values provided by CPF for C objects such as pointers and structures. Scalars, which are not distinguished by the policy, are all represented using a single abstract value, $\$\text{scalar}$ (an exception is that 0 is recognized in some situations, since it serves as NULL in C). For pointers that are null (i.e., that may be assigned null), a special memory location, $\text{nullLoc}$, is used. Pointers that are not null (i.e., that must point to a non-null memory location, albeit one that may contain garbage) point to actual, non-null locations in the CPF C memory model. Two type annotations, $\$\text{null}$ and $\$\text{notnull}$, are provided to allow developers to encode this information, with unannotated pointers considered $\$\text{null}$. Annotation $\$\text{nullity}$ provides additional support for stating or checking null properties of pointers and can be used with logical connectives to form more complex expressions.

Given these abstract value and annotation definitions, the next step is to define the policy semantics used during analysis. NOTNULL includes semantics for expressions and for two kinds of statements: conditionals and loops. Also included are semantics for pointer allocation, to ensure that null and notnull pointers are initialized correctly. Some of the expression rules for pointer dereferencing and assignment are shown in Figure [8.21](#). The dereferencing

\[
\langle k \rangle * (\text{ptr}(L)) \ldots /k \quad \text{if } L \neq \text{nullLoc} \quad (8.22)
\]

\[
\langle k \rangle \quad * (\text{ptr}(\text{nullLoc})) \ldots /k \quad (\text{env}) \quad \text{Env}/env \quad \text{issueWarning}(1, msg1, Env) \land \text{fail} \quad (8.23)
\]

\[
\langle k \rangle \quad (\text{ptr}(\text{nullLoc})) \rightarrow X \ldots /k \quad (\text{env}) \quad \text{Env}/env \quad \text{issueWarning}(1, msg2, Env) \land \text{fail} \quad (8.24)
\]

\[
\text{assignAllowed}(\text{ptr}(\text{nullLoc}), L, (\text{Env}/L, \ldots, \text{dsl}(\text{DSL}) \text{dcl}(D) \ldots)) = \text{false}
\]

\[
\text{if isPointerDeclarator}(D) \text{ and DSLContainsTAnn}(\text{DSL})
\]

\[
\text{and @cpf}(N) := \text{getDSLTag}(\text{DSL}) \text{ and isNNNotNull}(NN) \quad (8.25)
\]

Figure 8.21: Not Null Policy: Pointer Dereferencing and Assignment
int *p = NULL;
if (p) {
    ... *p = 5; ... /* Should not cause a warning */
}
*p = 10; /* Should cause a warning */

Figure 8.22: Not Null Policy: Avoiding False Positives

operator, *, is strict, causing $E$ in $*E$ to be evaluated using heating and cooling rules. Once $E$ is evaluated, one of Rules (8.22) and (8.23) could apply. Rule (8.22) handles the case where $E$ evaluates to a pointer to location $L$, where $L$ is not nullLoc; in this case, we then look up the value at location $L$ using operation $\text{llookup}$. Rule (8.23) handles the case where the program attempts to dereference a null pointer, and yields an error, issued using $\text{issueWarning}$, with $\text{msg1}$ standing in for the issued warning message. Rule (8.24) defines this same rule for pointers to structures, where field $X$ is being accessed using the $->$ operator (like $p->x$). Both failure cases return a special $\text{fail}$ value, which is then propagated through other expressions without causing additional error messages to be generated. Finally, Rule (8.25) is used during assignment to determine if an assignment is allowed. This rule represents the failure case: when assigning a value that is a null pointer, if the location being assigned to ($L$) is declared to be a pointer ($\text{isPointerDeclarator}$), and if the declaration includes a type annotation ($\text{DSLContainsTAnn}$, for declaration specifier list contains type annotation), and if that annotation is $\text{nonnull}$ ($\text{isNNNotNull}$), reject the assignment. This catches attempts to assign null pointers to objects annotated as not null.

For declarations, the pointer allocation semantics take account of the annotations on the pointer declaration, allocating storage for pointers annotated as $\text{nonnull}$ and assigning $\text{ptr(nullLoc)}$ to pointers either not annotated or specifically annotated as $\text{null}$. The statement semantics for conditionals and loops are also specialized for NOTNULL, specifically to account for specific checks against null in the conditional or loop expression. If the expression is a check to ensure that a pointer is not null, we can assume that it is not null in the then branch or loop body, but that it is null in the else branch, eliminating common false positives. An example code fragment is shown in Figure 8.22.

Here, the assignment of 5 to $*p$ should not cause a warning to be issued, since the user explicitly checks that $p$ is not null, while the assignment of 10 to $*p$ should still cause a warning.

To allow for this scenario, the conditional logic is overridden, with specific checks for different expression “patterns” common in null checking – the use of just the pointer, like in $\text{if}(p)$, or the use of a comparison against $\text{NULL}$, like $\text{if}(p \neq \text{NULL})$ or $\text{if}(\text{NULL} \neq p)$. In both patterns, the conditional body will only be entered when the pointer is not null, so it is possible to assume this at
```c
#include <stdlib.h>
int f($notnull int *v) {
    int x,*p;
    int $notnull *q;
    p = (int*)malloc(sizeof(int));
    q = v;
    q = p;
    *p = 5;
    if (NULL != p) {
        *p = 10;
        x = *p;
        free(p);
    }
    return x;
}
```

Figure 8.23: NotNull Example

the start of the body. Similar logic works for loops and if statements with else branches. Note that we should not try to do the opposite, and assume a known not null pointer is null after a check like if(!p), since this could lead to a false positive.

Figure 8.23 shows an example of using the NOTNULL policy, with Figure 8.24 showing the analysis results. Function f takes a single parameter, an integer pointer declared to be $notnull, and then declares three variables, an integer x, a (potentially null) integer pointer p, and a $notnull integer pointer q. p is allocated on line 5 using a call to malloc; since malloc returns null when no memory is available, p could still point to null. Line 6 shows a correct assignment, of one $notnull pointer to another. Line 7 generates an error, though, since it could set q to null. Line 8 also generates an error, since p may still be null when it is dereferenced. Line 9 contains an explicit check to see if p is null; since the body is only executed when p is not null, we can assume this at the start of the body, which is why lines 10 and 11 do not generate error messages.

8.7 Discussion

Chapters 7 and 8 presented policy frameworks, including an extended description of the C Policy Framework. This section discusses some of the limitations of this work; a comparison with other related work is provided in Chapter 9.

First, while the analysis semantics used as part of the policy framework are often based on an existing semantics of the language, there is no automatic way to either generate the analysis semantics or verify that the analysis semantics are

ERROR on line 7(1): Attempting to assign possibly null pointer to another pointer annotated as definitely not null.

ERROR on line 8(1): Attempt to dereference possibly null pointer.

Figure 8.24: NotNull Sample Run
correct with regards to either the static or dynamic semantics of the language. Work on this, in the context of generating environments for program verification based on a dynamic semantics of the language, is ongoing.

Second, currently the focus has been on reuse of definitions of language features across multiple policies within a language, instead of across policy frameworks for different languages. This is in contrast to analysis domains, which are intended to be reusable both across languages and across different policies. The work on the module system is designed to support language feature modules for analysis semantics as well, so we believe that it will be easier to construct the language-specific parts of a policy framework from reusable pieces. However, this area still needs further work.

Third, while the use of Maude provides many advantages, it can be an unfamiliar platform for developing analysis policies for many potential users, both in terms of the semantic techniques being used and the underlying (rewrite-based) platform. One potential solution is to develop a layer on top of Maude for developing analysis rules, with an automatic translation into Maude generating the analysis semantics. This would still require some knowledge of Maude in cases where something does not work as expected, but could provide a friendlier environment both for those that are defining policies and for those reviewing the policies to understand the analysis being performed.
Chapter 9

Related Work

Programming language semantics has been an active area of research since at least the 1960s, with a number of different formalisms, including supporting tools and techniques, defined between then and now. The first section in this chapter, Section 9.1, compares a number of these formalisms with K. To maintain focus, the formalisms presented here have been selected based either on their popularity, on being especially relevant to the design goals of K and the work presented in this thesis, or both. We especially focus on modularity, one of the motivations for K and a driver for much of the work outlined in prior chapters; tool support is also discussed briefly, with short evaluations and comparisons of a number of popular tools built to work with language definitions.

Section 9.2 then focuses on program analysis, especially in the context of systems that use either type or code annotations. The exception to this is a comparison between the CPF UNITS policy and units analyses devised in other frameworks, including solutions that make use of annotations, APIs for unit manipulation, and language features specifically targeted at units (or features which can easily be retargeted towards units). This provides related work and comparative information both for the concept of policy frameworks as a whole, and for one specific instantiation of a policy framework, the CPF UNITS policy, that has driven much of the work on policy frameworks.

9.1 Programming Language Semantics

A number of techniques for defining the semantics of a programming language have been developed over the years. The two most popular styles of semantics for defining language features (instead of for verifying properties of programs) have been operational semantics and denotational semantics. Various specific instances of these styles are discussed below, including: structural operational semantics, natural semantics, modular structural operational semantics, the Scott-Strachey style of denotational semantics, and monadic semantics.

Although operational and denotational styles have been the most popular, other semantics techniques, such as action semantics, rewriting logic semantics, and abstract state machines, have also been used to define programming languages. These are also discussed further below, along with a new style of
semantic definition, component-based semantics, that provides a way of structuring modular language definitions to improve the reuse of individual language features.

For each form of semantics discussed in this section, an attempt has been made to provide enough information to motivate the comparison with K and to provide intuition as to where tool support would be most helpful. For those interested in learning more about the techniques discussed here, specific references that provide more detailed explanations are cited as part of the discussion. Good overviews, covering multiple semantic techniques, can be found elsewhere \[164, 136, 146\], including detailed comparisons between these methods and both rewriting logic semantics \[160\] and K \[155\].

### 9.1.1 Operational Semantics

K takes an operational approach to defining programming languages, with rules indicating transitions between configurations. This is the same approach taken in two popular styles of operational semantics: structural operational semantics, or SOS \[152\], also referred to as small step semantics or transition semantics; and natural semantics (NS) \[97\], also referred to as big step semantics.

In SOS, configurations include the program and, if needed, the program state. The program state includes information such as the environment, the store, definitions of classes and methods, etc. Semantic rules, of the form $c \xrightarrow{l} c'$, are given to define the language semantics; $c$ and $c'$ are configurations and $l$ is an optional label. The rules are defined to capture individual steps of computation, such as a step of evaluation of a single operand in an arithmetic operation. As an example, the rules for addition in a simple expression language without state are shown below:

\[
\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2} \quad \text{(EXP-PLUS-L)}
\]

\[
\frac{e_2 \rightarrow e'_2}{n_1 + e_2 \rightarrow n_1 + e'_2} \quad \text{(EXP-PLUS-R)}
\]

\[n_1 + n_2 \rightarrow n, \text{ where } n \text{ is the sum of } n_1 \text{ and } n_2 \quad \text{(EXP-PLUS)}\]

Note that the left operand, $e_1$, is evaluated first, until it evaluates to a number, $n_1$. Once $e_1$ evaluates to $n_1$, $e_2$ is evaluated to $n_2$, enforcing a left to right order of evaluation. Once both operands have been fully evaluated their sum is computed, yielding $n$, the sum of $n_1$ and $n_2$.

In natural semantics \[97\], configurations are similar to SOS configurations. However, the rules, instead of capturing an individual computational step, are designed to completely evaluate the language feature being defined. An equivalent rule to the above, given for natural semantics, is shown below:
As in the SOS example, \( e_1 \downarrow n_1 \) and \( e_2 \downarrow n_2 \), with \( n \) then calculated as the value of \( n_1 + n_2 \).

**Comparison:** SOS and NS both have several advantages over K. First, language definitions given in either can be fairly mechanically turned into language interpreters. This is especially true of NS, where the semantics can be represented using recursive function calls and pattern matching. The natural model for K interpreters, as term rewrite systems in Maude (or other systems that support both associative and commutative matching), generally has slower performance, which is one of the motivations behind still early work on generating high performance interpreters and compilers from K definitions [78].

Second, SOS and NS both also tend to stay closer to the abstract syntax of a language, providing a close mapping between syntax and semantics. K definitions quite often introduce intermediate operations to represent different steps of computation, which, if not done carefully, can make definitions harder to understand. This has been remedied somewhat in K by using strictness attributes, allowing a number of the rules used to evaluate individual operands of a language construct to be generated automatically.

Third, although some work on using a K semantics as the basis for formal proofs of both programs and programming language meta-theory has begun [44, 158, 157], work on using operational techniques for these proofs is currently more advanced, with a wider array of related work and techniques that can be referenced by practitioners.

K also has several advantages over both SOS and NS. First, neither SOS nor NS are modular, with changes to the configuration requiring changes to all rules in the semantics, even those rules that just propagate the added configuration items. For instance, if the simple expression language used in the SOS rules shown above is extended with an environment to support the use of variables, one would generally change the rules as follows:

\[
\frac{\langle e_1, \rho \rangle \rightarrow \langle e'_1, \rho \rangle}{\langle e_1 + e_2, \rho \rangle \rightarrow \langle e'_1 + e_2, \rho \rangle} \quad \text{(EXP-PLUS-L)}
\]

\[
\frac{\langle e_2, \rho \rangle \rightarrow \langle e'_2, \rho \rangle}{\langle n_1 + e_2, \rho \rangle \rightarrow \langle n_1 + e'_2, \rho \rangle} \quad \text{(EXP-PLUS-R)}
\]

\[
\langle n_1 + n_2, \rho \rangle \rightarrow \langle n, \rho \rangle \text{ where } n = n_1 + n_2 \quad \text{(EXP-PLUS)}
\]

Unfortunately, the rules are not changed just when going from a semantics with no program state to one with program state. Instead, rules are continually adapted to include new state components, even if these components are not used.
in the rules (as they were not above). This makes it much harder to use SOS or NS to give a semantics to large languages, where it would be advantageous to build the semantics in pieces. It also makes both frameworks poor choices for language prototyping or for developing reusable language feature definitions, since both would require the rules to be adapted (maybe repeatedly) to match changes to the configuration.

Second, K supports a richer model of concurrency than either SOS or NS. SOS supports an interleaving semantics, where a concurrent execution is represented as multiple sequential executions, each of which is one of the possible interleavings of computational steps that could be taken by the concurrently-executing processes. The model supported by NS is simpler: given that each rule in the semantics is atomic, taking the language construct to its value in one “big step”, only one of the concurrent paths is chosen, with interleaving not allowed (e.g., given a number of threads, each thread would run to completion before the next could start, with no communication between the two allowed except when one yields a result that another then uses).

Some research has tried to work around this by adding a trace relation to natural semantics [125]. This allows some concurrent features to be captured, with a main thread that computes a value and worker threads that can communicate with the main thread asynchronously, influencing the final computed value. However, this model is still limited: interleavings are lost, since only the computed value is deemed important, not the process used to compute this value; worker threads cannot communicate with one another; and it is still difficult to represent situations such as divergence and deadlock (especially of the main thread). Because of this, concurrent languages defined in natural semantics “escape” to some other form of semantics when defining the concurrency features [11][174]. The author is unaware of any real concurrent languages defined completely using natural semantics.

In contrast to both SOS and NS, K supports true concurrency [129][128][117], which allows defining semantic rules where multiple concurrent processes take computational steps at the same time. This provides a semantics closer to the actual behavior of programming languages than an interleaving-based semantics, especially with the increasing use of multi-core, parallel, and distributed systems that have actual concurrency, instead of just systems where concurrency is simulated (for instance, by thread scheduling).

Third, both SOS and NS have difficulty representing language features that involve complex control flow. Features like exceptions generally require either modifying the configuration or passing program values representing the control flow event (like a thrown value), with special versions of rules needed to either check this flag or propagate this event; some features, like continuations, have no natural definition in either SOS or NS, but instead require defining the semantics of both the language and at least part of the evaluation mechanism (i.e., defining a runtime that supports continuations, versus defining the language directly).
This has led to the use of conventions in actual language definitions, such as those used in the definition of Standard ML to handle exceptions [124]. By contrast, these features can be modeled easily in K because of the explicit representation of the current computation as a term, which allows computations to be stored, restored later, or constructed as necessary.

**Tool Support:** A classic example of a semantics-based language environment is Centaur [20], an ambitious project which allowed languages to be defined formally, with the formal definition then used to generate a suite of graphical language-support tools. Language semantics were defined using natural semantics in a Prolog-like language named TYPOL; this TYPOL definition was then converted into Prolog, allowing programs to be evaluated in the semantics using logical reasoning. Centaur inherits the benefits and faults of natural semantics, including the lack of modularity, making it difficult to create reusable definitions of language features. One goal of our work on K is to create tools like those provided by Centaur; the K tool support currently focuses on program execution and analysis, instead of generating tools, such as editors and debuggers, which would be useful for language designers and language users.

Two other tools used for operational definitions are LETOS [70] and RML [151]. LETOS provides a lightweight language development environment for operational semantics definitions in a literate style reminiscent of literate Haskell, with LaTeX formatted documentation and specially demarcated blocks containing language definition rules. These rules can be translated to allow program execution in either Miranda or Haskell. LETOS can also be used for denotational definitions, but with reduced functionality, mainly losing the ability to visualize rule application. RML is focused mainly on compiling natural semantics definitions to generate efficient implementations, but lacks features such as visualization of rule application available in some other tools. Both tools again inherit the limitations of their formalisms, but both also provide an ability currently lacking in K: the ability to generate interpreters in languages which can then be compiled for performance.

The Ott system [161] is described as a meta-language and tool designed to support the “working semanticist”. The meta-language allows definitions of languages to be specified as inductive relations, with special support for binding and substitution. Tool support includes the ability to include LaTeX documentation in definitions, check definitions to ensure they are consistent (in the words of the authors, to “sanity-check” the definitions), and generate language definitions and proof tasks for a number of commonly-used theorem provers: Coq, HOL, Isabelle, and Twelf. Ott has been used to define a handful of languages and language calculi, including a significant subset of OCaml and the Lightweight Java Module System [172]. Ott’s strongest aspect is its focus on theorem proving; the lack of executability, and the reliance on non-modular language definition techniques, would make it difficult to use as a language design environment.
and would make it hard to reuse parts of language definitions, something that could be quite useful in a theorem proving context. With these limitations, it is unclear how easy it will be to define an entire language in Ott or to extend existing language definitions as languages evolve over time.

Reduction Semantics

Reduction semantics [50] was designed to provide better support for defining control-intensive features in (especially imperative, but also functional) programming languages [49]. This is done by defining reduction contexts, which are places in a program where computation can occur; and reduction rules, which determine steps of program evaluation. Support for control-intensive features is provided by allowing the rules to access not just the part of the program inside the reduction context “hole” (i.e., the redex), but also the surrounding context of the computation, which can be saved, modified, or replaced as needed. The ability to manipulate the computational context, and the focus on reducing terms in a context, gives rise to two other common names for this style of semantics: context reduction, and context-sensitive term rewriting.

Comparison: Context reduction maintains some of the same advantages of SOS over K, such as the close relationship between the syntax and semantics, while also providing improved support for defining control-intensive features. Work on using context reduction for language meta-theory proofs, such as proofs of type soundness, is also more advanced [193] than the equivalent research using K [44]. Finally, the ability to abstract the context in which a rule applies provides some of the same modularity benefits as the use of matching over multisets in K or the use of labeled configuration items in MSOS (discussed below), insulating rules from changes not related to the referenced parts of the configuration.

At the same time, context reduction also has some limitations, both compared to other operational styles and to K. First, the execution model for context reduction definitions is quite complex, with the system needing to find a redex at each step, requiring the entire term tree to be searched. Certain techniques, such as refocusing [37], help to alleviate this, but only work in restricted cases. For instance, refocusing requires that the unique decomposition property holds, which states that any non-value term $t$ can be uniquely decomposed as $t = C[r]$, where $C$ is the context and $r$ is a potential redex. However, this property only holds in deterministic languages, since it states that there is only one part of the program that can be evaluated (i.e., run) at any one point in time.

Second, reduction semantics only supports an interleaving semantics for defining concurrent language features, giving it a more limited concurrency model than the true concurrency model supported by K and rewriting logic. Finally, although the representation of context provides some modularity, changes in context, such as adding new context “groupings” to represent threads, would
still require changes to rules that referenced the prior context layout. This limits
the ability to reuse defined language features and to extend existing languages
with new language features, operations directly supported using the modularity
features (context transformers, the module system) of K.

**Tool Support:** PLT-Redex [115] is a graphical environment, built on top of
PLT-Scheme and the DrScheme interface, that allows for the development of
language definitions using context reduction. PLT-Redex includes several useful
features which have served as an inspiration as we have designed tool support
for K, including the ability to visualize reduction steps graphically, design test
suites for semantic features, and generate some forms of documentation for the
semantics.

However, PLT-Redex does have some limitations which could make it difficult
to develop and experiment with large language definitions. One important
limitation is performance: language definitions run very slowly, and often have
a large resource footprint. This can make running even small programs very
time consuming. Part of this is due to the lack of definitional constructs, like
sets, commonly used in K definitions (all lookups become list lookups in PLT-
Redex). Another is that language definitions make use of Scheme, requiring an
understanding of the Scheme semantics to get a full, formal understanding of
the semantics being defined, a problem similar to that mentioned below with
semantics-based interpreters.¹ On the other hand, PLT-Redex has found a
number of applications in language design, spawning a local workshop and a
book [48] with a number of case studies.

**Modular Structural Operational Semantics**

Although SOS definitions have many benefits, one of the key problems, mentioned
above, is that the definitions are not modular, making it difficult to initially
construct and then maintain definitions of realistic programming languages. One
attempt to solve this problem is Modular Structural Operational Semantics
[135, 137], or MSOS. The key difference between MSOS and SOS is in the
use of labels on rules. In SOS, labels are generally not used in definitions of
programming language features, but are instead used for giving semantics to
other systems, such as process calculi. This has been exploited in MSOS [138]
by using the rarely used labels to hold all information from the configuration
except for the current program. Using certain notational conventions, it is then
possible to define rules that only refer to the parts of the state, given in the rule
label, that they need, with the others elided. This ability to not mention parts
of the state that are not used is key to ensuring modularity – as long as only the
unused part of the state changes, the rules given in the semantics do not need to

¹PLT-Redex could actually be considered to be a semantics-based interpreter, but, since it
provides a number of constructs for language definition that sit on top of Scheme, it is useful
to look at it separately.
change.

For example, the following rules give the MSOS definition of the plus expression, given above for both SOS (with and without state) and NS (just without state):

\[
\begin{align*}
    e_1 \xrightarrow{\ldots} e_1' & \quad (\text{EXP-PLUS-L}) \\
    e_1 + e_2 \xrightarrow{\ldots} e_1' + e_2 \\
    e_2 \xrightarrow{\ldots} e_2' & \quad (\text{EXP-PLUS-R}) \\
    n_1 + e_2 \xrightarrow{\ldots} n_1 + e_2' \\
    n_1 + n_2 \xrightarrow{\ldots} n, \text{ where } n = n_1 + n_2 & \quad (\text{EXP-PLUS})
\end{align*}
\]

Again, note that most of the information in the configuration is gone (or, more accurately, hidden in the label)– these rules now look very close to those given originally in SOS for a language with no state. The \“\ldots\” over each arrow represents unmentioned parts of the state, with \“\ldots\” representing the same state both above and below the line dividing the premise and conclusion of the rule. This can be read as \“if the transition from \(e_1\) to \(e_1'\) (for instance) makes some change in the state, then the transition from \(e_1 + e_2\) to \(e_1' + e_2\) makes the same change in the state\”. Because individual parts of the state are not named, and because changes to this state are still propagated, the state itself can change (say, when adding environments or updatable stores with assignment expressions) without requiring any changes to these rules.

In cases where the state should not be changed by a rule, it is possible to note this using \“\ldots\”, instead of \ldots, over the arrow. Technically, this makes the label unobservable, so the rule can make no changes to the state that would be visible elsewhere. Using \“\ldots\” is obligatory for axioms, to ensure that they do not allow arbitrary changes to the state. It is also possible to use \“\ldots\” for rules with premises, which can be important in some contexts (such as for some proofs), but which could also limit reuse of the rule, as it could no longer propagate changes to the state not mentioned directly in the rule. An example with \“\ldots\” is shown above in the axiom given for plus expressions.

When the state is needed, the state components in the label are treated similarly to components of an ML-like record, allowing them to be accessed by name. A rule for name lookup could be:

\[
x \xrightarrow{\text{Env}=\text{env},\ldots} \text{env}(x) \quad (\text{LOOKUP})
\]

To represent changes from the initial information in the state, components of the state can be given with a prime, representing the state at the end of the rule. For instance, a rule in an imperative language for assignment, where the state contains an environment (mapping names to locations) and a store (mapping locations to values) would be:
\[ x := v \xrightarrow{Env=\text{env}, Store=\text{mem}, Store'=\text{mem}[\text{env}(x)\leftarrow v],} \text{skip} \] (\text{ASSIGN})

where \( \text{mem}[\text{env}(x) \leftarrow v] \) is the original value of \( \text{mem} \) but with the value at location \( \text{env}(x) \) replaced with \( v \).

One shortcoming of MSOS is that the rules, with configurations given on labels, look more complex, or at least unfamiliar, to those familiar with standard SOS rules. This has been addressed in Implicitly-Modular SOS (I-MSOS) [144], which allows the use of the more familiar SOS notation when defining rules. I-MSOS rules are then automatically translated into MSOS rules, providing both the simplicity of SOS and the added modularity of MSOS. An example, drawn from a recent paper [143], provides an example of the rules used to evaluate a let construct:

\[
\begin{align*}
\rho \vdash E & \rightarrow E' \\
D & \rightarrow D' \quad \text{(LET-DECL)} \\
(\text{let } D \text{ in } E) & \rightarrow (\text{let } D' \text{ in } E) \\
\rho'[\rho_1] & \vdash E \rightarrow E' \quad \text{(LET-EXP)} \\
\rho \vdash (\text{let } \rho_1 \text{ in } E) & \rightarrow (\text{let } \rho_1 \text{ in } E') \\
(\text{let } \rho_1 \text{ in } V) & \rightarrow V \quad \text{(LET-FINAL)}
\end{align*}
\]

The shading in the definition of the relation specifies which parts of the definition represent the state \( (\rho \vdash) \) versus which parts represent the syntax \( (E \rightarrow E') \). It also provides the information needed to transform the rules into equivalent MSOS rules. Note that, of the three rules, only the second mentions \( \rho \), making it implicit in the other two.

**Comparison:** MSOS and I-MSOS are both extensions of SOS targeting SOS’s modularity problems. Thus, they maintain the advantages of SOS over K (except, perhaps, in program proofs and language meta-proofs, where MSOS and I-MSOS have not been as widely used), while also addressing one of the major disadvantages, modularity. MSOS and I-MSOS provide the same modularity as K with context transformers, allowing rules to continue being used unchanged as a language evolves, and also allowing rules to be reused across languages.

Since MSOS and I-MSOS (from here just MSOS, unless the distinction is important) are built on top of SOS, they also share some of the same disadvantages. MSOS uses an interleaving semantics, versus the true concurrency model used in K, and continues to have some of the same challenges in defining control-flow intensive features, especially call/cc. It is, however, now much easier to add support for configuration “flags”, such as a flag indicating that a halt command has executed, since only the top-level rule (such as statement sequencing) and the rule that sets the flag need to be aware of this.
A final distinction is that the configuration itself is much more dynamic in K than in MSOS: it is possible to rename cells or add new cells in rules where, to the authors knowledge, it is not possible to do something similar in MSOS. While not essential in many programming languages, this can be useful in the definition of certain features. For instance, CPF, described in Chapter 8 regularly adds and removes cells during program analysis, based on the needs of the rules in the analysis semantics. As another example, KOOL, described in Chapter 3 renames the cell holding the store during garbage collection (described in Chapter 6), indicating that none of the rules that match the store can apply. This makes it easier to guarantee that the mutator and the collector do not conflict. Also, most concurrent languages defined in K create new cells to represent new threads (and their contents). This dynamism comes with a potential cost: verifying the correctness of both language features and programs in defined languages may be harder with a dynamic configuration, and would make formalization using standard techniques (such as giving the configuration as a tuple) more challenging.

**Tool Support:** There have also been several tool support platforms introduced for MSOS during its development. One method of executing MSOS definitions that has been explored is to use Prolog [139], with evaluation then based on Prolog reasoning. Another alternative, the Maude MSOS tool [27], is an extension to Maude built using Full Maude [41, 33]. The Maude MSOS tool allows MSOS-specific Maude modules to be defined using MSDF, the modular SOS definition formalism. These modules are then automatically translated into standard rewriting logic system modules using a semantics-preserving transformation [118] from MSOS to rewriting logic, allowing MSOS rules to be used for program execution and analysis. The K module system provides similar functionality, leveraging the K tool kit to provide a translation into Maude while also providing an additional layer of abstraction, allowing use of a friendlier syntax for defining modules as well as syntactic sugar for a number of common scenarios, sort aliasing and variable prefixes being just two examples. This additional layer of abstraction has proven quite handy in working around some of the parsing restrictions in Maude: for instance, because of the way that rules and equations are parsed, variable prefixes are quite challenging to add directly into the Full Maude version of the module system that is currently being built, something the author unfortunately discovered in the middle of the implementation.

The Maude MSOS tool not only supports standard monolithic definitions, but also supports the new component-based style of definition. This definitional style, along with other work on tool support for MSOS (which also supports this style), is discussed below.
9.1.2 Denotational Semantics

In denotational semantics, the semantics of a language construct is given as a function which maps the construct to its *denotation*, a mathematical value that provides the meaning of the construct. In the Scott-Strachey approach to denotational semantics \[170, 159, 132\], semantic functions map each language construct to a value in a Scott domain \[66\], formed using both primitive values (e.g., natural numbers) and constructs such as function spaces, products, and sums. The semantics are compositional, meaning that the semantics of a feature is defined in terms of the semantics of its subfeatures.

As mentioned above, K takes a more operational approach to language semantics. However, a slight change in perspective allows K definitions to be seen as being denotational as well. For deterministic languages, K language definitions can be translated into equational logic theories (i.e., rewriting logic theories with no rewrite rules), which can be given an algebraic denotational semantics \[121\], the preferred method being through the use of initial algebra semantics \[83\]. In initial algebra semantics, programs are represented as algebraic terms, which are members of (from Chapter 2) the term algebra \( T_\Sigma \). \( T_\Sigma \) is *initial* in the class of all \( \Sigma \)-algebras, meaning there is a unique homomorphism from \( T_\Sigma \) to any other \( \Sigma \)-algebra. This homomorphism can be seen as the meaning, or program evaluation, function, taking the syntax of the program to its denotation, a value in one of the carriers of the target algebra. In the case of equational logic, the denotation is actually an equivalence class, made up of all terms which can be shown to be provably equal using the stated axioms, given as equations, and the rules of equational deduction (given in Chapter 2). For programs that terminate, the final value (i.e., 3) of the program will be one of the terms contained in the equivalence class including the initial program (i.e., a program is considered equivalent to its denotation).

For languages with nondeterminism, including concurrency, standard denotational definitions use the concept of *powerdomains*, which provide a powerset-like construct for domains, capturing the fact that a program can produce different answers based on differing underlying orders of execution. K also supports a form of denotational semantics for languages with nondeterminism through rewriting logic using the initial model semantics of rewriting logic \[117\]. In this semantics, the model of a program is given in terms of a category, where the objects of the category are equivalence classes of terms (where terms are equivalent when provably equal using the rules of equational deduction). In rewriting logic, *rules* are used to move between these equivalence classes, providing a way to explicitly capture the nondeterminism present in the language semantics. This is represented through the morphisms of the category, which are also equivalence classes of terms, but in this case terms based on the defined rewrite rules, the rewriting logic rules of logical deduction, and several additional axioms.

**Comparison:** The standard techniques for denotational semantics provide
several advantages. The close link between denotational semantics and functional languages, especially pure functional languages such as Haskell, provides a natural way to “implement” denotational definitions, while also allowing techniques developed for semantics (like monads, discussed below) to filter into standard practice in the functional language community. The abstraction gained from using denotational definitions has also proven to be valuable for better understanding and reasoning about programs and programming language definitions. This can be seen in work such as the LOOP project [92, 175], which used a denotational definition of the JavaCard version of Java as a basis for verifying the correctness of JavaCard applications. Finally, the use of lambda notation in the definitions of languages provides a standard base language for describing and comparing language features.

There are also some disadvantages to these techniques which are addressed by K using the underlying algebraic denotational semantics and initial model semantics of rewriting logic. Initial algebra semantics provides a more natural mapping from the abstract syntax of a program to the semantics, without a need to encode language semantics into potentially complex lambda terms. Also, the first-order nature of the definitions can occasionally provide more insight, since operations provided by the semantics are not given in terms of features of the underlying notation which may themselves have complex definitions (such as variable binding and substitution). Finally, rewriting logic provides better support for reasoning about and representing concurrency, without the need to use constructs such as powerdomains (for which, to this author’s knowledge, there are still several competing definitions).

Monads

A well-known limitation of denotation semantics is that, like the definitions of language features in structural operational semantics, denotational definitions of language features are not modular [132]. The use of traditional lambda notation to define the semantics imposes the same restrictions found in pure functional languages: any state, such as (in semantics definitions) environments, stores, etc, needs to be threaded through the defining functions. When new language features are added that require extensions to the configuration, all existing rules need to be modified, even those that do not use the new parts of the configuration, to ensure the entire configuration is propagated.

One proposed solution for improving the modularity of denotational definitions is monads [126, 127]. Although quite different from the solution used in MSOS, there are also some similarities. In MSOS, configuration items are given as part of the labels on transitions, providing what can be viewed as an extensible record structure for storing configuration information. Using monads, this configuration information is instead stored inside a monad which represents

\footnote{Note that this could be seen as a disadvantage by some, since this can require more formalization effort.}
a computation. The semantic functions return computations, not values, allowing computations to be threaded through the semantic rules. At the same time, the monad itself is treated like an abstract data type, with an interface that knows about the configuration items stored inside and exposes them through a defined (functional) interface. In essence, instead of threading the configuration through the functions in pieces, it is automatically carried inside the computation, allowing rules that know about the monad to be defined without then needing to be changed when the internals of the monad change (assuming the interface does not also change).

Although introduced as a method to modularize denotational definitions, monads were popularized when first applied in a more practical setting: representing state in purely functional programming languages such as Haskell, especially in the context of language interpreters and compilers [186, 187]. This initial work provided a way to localize stateful operations, but did not provide an easy way to either combine different features or “plug in” features without changing the surrounding code. Subsequent work, [103, 95, 160, 108, 107], while often maintaining a focus on interpreters, also focused on methods of programatically combining monads using monad transformers, ensuring that the resulting monad (the combination of the input monads) preserved the required mathematical properties. The result of one line of this work is Modular Monadic Semantics [107], which structures the semantics of a language by defining them in terms of a number of predefined monads that provide common language building blocks (stores, continuations, etc).

Another line of work has focused more on the theory surrounding monads, although it has also (in many cases) maintained a practical focus, with implementations of the concepts provided in functional languages such as Scheme, Haskell, or Gopher. This includes work on language development environments such as Semantic Lego [45], which focused on combining monads for different language features using a concept called stratification, allowing monads to be combined by “building them up” in the desired order, instead of “lifting” them through other monads with monad transformers. Even with stratification finding general-purpose ways to combine monads has still been problematic; some recent work has focused on defining the combination of two monads as their coproduct [111], which appears to work in most cases except for continuations, which are problematic for many of the proposed methods.

Finally, along with some of the work mentioned above [107], some additional work on monads has focused on issues that are more specific to compilers [69, 162, 163], such as the need to divide compilation up into different stages (represented by a monad for each stage).

**Comparison:** Monadic semantics provides a structuring mechanism for denotational semantics, so it shares many of the same benefits and limitations with standard denotational semantics definitions. The main distinction is that monads
provide a way to make languages more modular, with individual features designed around monads that contain the needed state. Monads have also been highly successful as a method for structuring functional programs, leading to a number of techniques for building language interpreters, using monads, in functional programming languages (especially Haskell and its variants and Scheme).

In comparison with K, there are also several limitations of monadic semantics. The main limitation, outside of those already mentioned for denotational semantics, is that there is no truly modular way to combine different monads using monad transformers. In some cases, there is no automatic way to meaningfully combine modules. Instead, transformers sometimes must be written by hand, ensuring that the correct monad laws hold (especially problematic, as usual, are continuations). Monad composition is also order dependent, meaning that a different semantics could be given to a language based on the order in which the underlying monads are combined. Since the number of transformer application orderings is quadratic in the number of monads being combined, this can quickly grow unwieldy in large language definitions. It also seems likely that, in some cases, it would not be possible to find an ordering that would satisfy all the needs of the language.

These problems are dealt with in K by using one global transformation over all the rules in the language once the rules are chosen and the language configuration is assembled, versus using a series of individual transformations as the language is built. This transformation should be able to build only one proper configuration; if it can build more that one, the rules are ambiguous, and the user is tasked with fixing the rules to ensure a unique final configuration can be found.

**Tool Support:** Most of the work discussed above was conducted in the context of creating modular interpreters, leading to several tools designed for this purpose. Some of the most notable include: interpreters based on pseudo-monads, written in Haskell [166]; monadic interpreters and compilers, written in Gopher (and, in the latter case, targeting Standard ML) [105, 107]; monadic interpreters written using Scheme [45]; and compilers built with explicit compiler stages, represented as monads [69, 162, 163].

An advantage of these tools over the current tool support for K is the use of more typical functional languages for implementations, which provides a more familiar environment and features such as compilation. A disadvantage, already mentioned above, is that the specification of the semantics of one language using another can mask the actual complexity of certain language features, and can require the language designer to understand both the defined and defining languages to fully understand the language definition. Otherwise, the benefits and limitations are as described above.
9.1.3 Action Semantics

Action semantics was defined with the goal of “allow[ing] useful semantic descriptions of realistic programming languages” [133, page xv]. It includes several major differences from standard operational and denotational styles of semantics, while still maintaining some of the signature features (definitions are given using an operational style, but are also compositional and map to denotations, albeit not given as values in Scott domains).

First, instead of using lambda notation (like denotational definitions), which can become quite complicated as language definitions become larger, more complex, and more realistic, action semantics uses action notation. Action notation defines a number of primitive actions (that perform computation) and action combinators (that combine smaller computations into larger computations), as well as methods for defining data and yielders, entities that can be evaluated to yield data. Although formally defined, action notation uses English instead of mathematical notation, making language definitions more readable.

Second, action semantics has improved support for modularity over standard operational and denotational semantics, giving it capabilities which appear similar to those of monads. Items in the configuration are modified by actions but propagate, for the most part, behind the scenes, using special actions to (for instance) allocate and update storage cells. Actions are separated into facets, each of which deals with a different kind of information (transient data, bindings, control flow, etc). Parts of a definition that work on one facet then do not need to worry about changes to other aspects of the definition. Much like MSOS, monads, or K, this protects definitions of language features from changes in unused parts of the state, allowing languages to evolve or features to be reused in other languages.

Third, action semantics includes improved support for concurrency over what is provided with denotational semantics [134]. This includes constructs to represent nondeterministic choices in the semantics (including from intentionally underdefined language features) as well as an asynchronous model of concurrency and distribution similar to that found in Actors [7]. This model supports true concurrency, with different agents making progress at overlapping times.

Originally action semantics was defined using a non-standard variant of SOS. The current definition instead uses MSOS, providing a cleaner underlying semantics and an opportunity to more easily extend action notation with new constructs if needed. Alternative definitions have been based around monads (described below as modular monadic action semantics) and ASMs (with an implementation using a tool and technique called Montages, presented below as part of the discussion around ASMs).

More recent work on action semantics has focused on defining languages using a number of small (per-feature) modules. This work is discussed below.

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3Some actions cover more than one facet.
under component-based semantics.

**Comparison:** Conceptually, there are some similarities between action semantics and K. In K, computations are driven by a number of provided and user-defined operators which work on the computation and other parts of the configuration. Action notation provides a similar language for action semantics, with semantics defined in terms of actions instead of K operations. In both K and action semantics there is no need to reference unused parts of the configuration, and both support true concurrency.

One difference between action semantics and K involves the use of kernel and derived operations. In K, the semantics of both kernel and derived operations is given in K notation, and, although a boundary between the two could be maintained, in practice it is not. In action semantics, kernel operations are defined directly in the underlying (now MSOS) semantics, while derived operations are defined in terms of other already-defined action semantics operations. This makes the boundary between the two more distinct.

Both approaches have disadvantages and advantages. In action semantics, it is harder to add new features not supported by the notation, since one needs to “escape” to MSOS to do this. However, the focus on defining new operations in terms of existing operations aids in understanding and reusing defined features and in comparing different definitions. In K, the ability to easily add new operations provides a straightforward way to support new language features, but the unwise use of this support can lead to redundant and harder to understand feature definitions. One purpose of the K module system is to provide a more direct way to reuse already defined features, introducing more discipline into the process of K language design.

Beyond this there are several other interesting points of comparison. First, in action semantics, the division of semantic features into different facets provides a separation of concerns in the semantics that does not exist in K and would be beneficial. Second, like in MSOS, it is not possible to define the semantics of call/cc in action semantics without defining the language semantics to also include the semantics of the underlying runtime. Third, action semantics supports some forms of concurrency – specifically, concurrency that can be modeled as agents communicating using asynchronous messages – but not others, including standard threading models with shared memory and threads that can be interrupted and resume execution later. K can naturally define these features.

**Tools:** ASD [152], the Action Semantic Description tools, provides an environment for working with action semantics descriptions. Included are tools for parsing descriptions, syntax-directed editing, checking for well-formedness, and transforming programs into equivalent programs using action notation in place of the original language constructs. ASD is written using the ASF+SDF Meta-
Environment [179], with ASD definitions translated into equivalent ASF+SDF constructs. ASD does not appear to support running programs using action notation, but instead is focused on working with language definitions. This is similar to the work on the K module system, although ASD supports features we are still adding (such as improved well-formedness checking of definitions, which is only partial at this point since much of the parsing of rules and equations is performed in Maude).

Another tool, the Maude Action Tool [38], is an action semantics-driven interpreter. It accepts a description of a language, defined in action semantics, and a program in the described language. The language description is translated from action notation into Maude notation, which is then used to “desugar” the input program into an equivalent program using action notation in place of the syntax of the language. Another transformation is used to transform the underlying MSOS semantics of action semantics into an equivalent rewriting logic theory (a system module). Using this rewriting logic theory, the desugared programs can then be executed in Maude. It would also be possible to apply Maude’s analysis and verification features (model checking, state space search), although it is not clear if this has yet been tried. This translation is similar in approach to the current K tools, although in K, instead of desugaring a program into semantic constructs, the semantics are built over the abstract language syntax.

There are several other notable tools for working with or interpreting programs using action semantics definitions. One [10] uses Montages [105] (discussed below with ASMs) to define the underlying semantics of action semantics, providing a semantics-based interpreter for running, debugging, and visualizing programs. Another tool, the Action Environment [178, 176, 177], is discussed below in the section on component-based semantics. Finally, Modular Monadic Action Semantics [189], which uses monads to implement the base semantics of action semantics and provides a monadic interpreter, is discussed later in this section.

Action semantics has also been a popular target for research on semantics-driven compiler generation. One example is OASIS [148], which takes a semantic description written in Scheme and generates a Perl-based compiler, which then generates executable code for SPARC processors. Another is Actress [23], which generates an action semantics-based compiler that then compiles source programs into C. Work on generating high-performance interpreters and compilers using K definitions is still in early stages of development [78].

Modular Monadic Action Semantics

One limitation of the initial version of action semantics [133] was that the underlying semantics were based on a non-modular SOS definition, making it challenging to extend action semantics with new facets or new basic constructs. Another was that the theory for reasoning about programming languages defined
using action semantics, as well as reasoning about programs based on an action semantics of the underlying language, had not been (and, to the author’s knowledge, still has not been) a focus of the research, making this kind of reasoning challenging.

One solution to this problem was to replace the underlying operational (SOS) definition of action semantics with a Modular Monadic Semantics (as discussed above), leading to a version of action semantics called Modular Monadic Action Semantics [188, 189]. The move to an underlying monadic semantics served two goals: first, it provided a modular base for defining extensions to action semantics; and second, it opened definitions up to modular reasoning techniques developed as part of the work on denotational semantics in general and monadic semantics in particular.

**Comparison:** MMAS provides some advantages over the original version of action semantics, both by making it easier to extend and (potentially) easier to reason about. An example extension involves adding support for continuations, a feature not supported in action semantics (at least without changing the nature of the definition to account for features of the language runtime). At the same time, this support comes at the cost of limiting the ability to represent nondeterminism and parallelism, something available in action semantics but problematic in monadic semantics.

With the new modular semantics underlying action semantics (through the use of MSOS), it appears that the first advantage, making it easier to extend action semantics, no longer holds. It is still not possible to define continuations, but other extensions should be possible. The second advantage, making language definitions and programs in the defined languages easier to reason about, may still hold, but the author is unaware of any work taking advantage of MMAS definitions for these purposes.

In comparison to K, MMAS does remove one disadvantage of action semantics, the inability to add features such as call/cc, but again only at the cost of weakening support for concurrency. Other advantages and disadvantages are the same as those given above directly for action semantics.

### 9.1.4 Component-Based Semantics

Component-based semantics [142, 143], referred to previously as constructive semantics [140], is a style of language definition that builds upon other modular formalisms, such as MSOS and action semantics. The main goal of component-based semantics is to define individual language features as reusable components, which can then be assembled (given proper tool support) into a complete language. A challenge is that most language features are initially defined in the context of a specific language, making it hard to reuse features in other languages that, even with the same semantics, may have a different syntax.
Component-based semantics approaches this problem by separating the concrete and abstract syntax, mapping the concrete language constructs to a number of language-independent abstract constructs (referred to in earlier work by the same authors as Basic Abstract Syntax [91]). Abstract constructs are defined over a number of sorts representing language constructs (like in K), with typical sorts like \texttt{Cmd} for command or \texttt{Exp} for expression. Examples of abstract constructs include \texttt{seq}, which sequences a list of commands; \texttt{bind}, which binds a value to an identifier; and \texttt{lookup}, which retrieves the value bound to an identifier [142]. The following example, taken from a recent paper [142], shows a mapping from a \texttt{while} construct that allows loop breaks to the related abstract constructs:

\begin{align*}
\text{Cmd}[[\texttt{while } E \ C]] & = \text{catch(\texttt{cond-loop(Exp}[E], \text{Cmd}[[C]]),} \\
& \text{abs(eq(breaking), skip))} & (9.1) \\
\text{Cmd}[[\texttt{break}]] & = \text{throw(breaking)} & (9.2)
\end{align*}

Rule 9.1 translates a while command, with loop expression \(E\) and body \(C\), into the \texttt{cond-loop} construct. This construct is surrounded by a \texttt{catch} construct, indicating that the loop may terminate abruptly; this is handled using the \texttt{breaking} handler, which does nothing (skips). Rule 9.2 then shows the semantics of \texttt{break}, which throws \texttt{breaking}.

The work on component-based semantics grew out of work on creating modular language definitions using both action semantics and MSOS. For action semantics, this work started with an initial module system for defining action semantics modules [39]. Each module is made up of a syntax and a semantics section, with the syntax section containing abstract syntax and the semantics section containing the action semantics definition of the language construct. Each module is focused on a specific feature, with more complex modules then built by either combining or extending other modules, eventually forming a language definition.

This work was then extended [40] to provide more structure to the modules, specifying three kinds of modules: semantic functions modules, declaring the names and types of semantic functions (similarly to declaring an operation in Maude); semantic equations modules, equivalent to the modules in the earlier work, providing the syntax and semantics of a single language feature; and semantic entities modules, providing action notation and auxiliary semantic entities. As in the earlier work, modules are built by either extending or combining existing modules.

This work was done directly using ASF+SDF, but managing the large number of modules created was problematic. Because of this, a new formalism for creating action semantics descriptions of single constructs, ASDF, was created [90].
along with tool support designed for working with ASDF (discussed below), has been used to define the semantics of Core ML [91].

**Comparison:** The definitional approach developed for component-based semantics should work for any semantic formalism which is sufficiently modular. At this point the main focus has been on using either action semantics of MSOS, but we believe that K would be an appropriate choice as well, since it provides the modularity needed for the component-based semantics technique to work.

The closest comparison, then, isn’t with K, but with the work on the K module system described in this thesis. It should be possible to use the K module system to support a component-based style, but that has not been done yet – instead, definitions make use of the abstract syntax of the language being defined. This limits reuse of defined language features to scenarios where, in the new language, the feature will have not only the same semantics but the same syntax as well. It also makes it more difficult to identify features which are shared by multiple languages, since differences in the syntax could “hide” this. Finally, the use of abstract constructs provides guidance about the appropriate size of a feature definition (i.e., one definition per module), which is recommended but not required by the module system.

However, component-based semantics also has some limitations. One is that the use of a translation from concrete constructs to abstract constructs may make it harder to understand a single language, even if it makes it easier to understand and compare multiple languages, since the features would be defined in a syntax further removed from that of the language. This translation can also make it harder to debug definitions, something that comes up quite often while defining complex languages, since errors can occur at both the semantics level and the translation level, and since it is necessary to run the translation “in reverse” to determine which language construct is the source of the problem. Finally, the use of one abstract construct per feature is realistic when defining just the standard static (types) and dynamic (execution) semantics of a program, but when semantics are also defined for program analysis and verification purposes, like in the work on Policy Frameworks discussed in Chapters 7 and 8, this requirement would cause a proliferation of names, and would also make it harder to define semantics as extensions to pre-existing named hooks (essentially, an abstract construct in the component-based semantics terminology), with the hook given different meanings by different analysis policies.

**Tool Support:** The Action Environment [178, 176, 177] supports the use of action semantics and the ASDF formalism for defining programming language features. ASDF definitions are translated to ASF+SDF, which also allows constructing a mapping from the concrete syntax of a language to its representation using abstract constructs. Tools provided as part of the Action Environment
include variants of the ASF+SDF tools, available through the use of the MetaEnvironment, as well as a type checker for action semantics functions and an action semantics interpreter. Tool support for working with ASDF definitions is more advanced than the current support for working with K definitions, although this is an area being actively worked on.

The Maude MSOS tool, described above, provided support for creating component-based MSOS definitions and executing them in Maude. It has also been shown that OBJ \[59\] can be used as a platform for experimenting with the component-based style of action semantics \[141\].

9.1.5 Other Semantics Techniques

A variety of other techniques for giving semantics to programming languages have been designed over the years. Included below are several that are especially relevant to K and to the work described in this thesis, but are not as popular as the operational and denotational techniques mentioned above. This includes Abstract State Machines; other manifestations of rewriting logic semantics; semantics-based interpreters; semantics of deterministic languages based just on the use of equational logic; and definitions of languages in theorem provers.

ASMs

Abstract State Machines (ASMs) \[88, 165, 18\] can be regarded to some extent as a “simplified programming language” with programs consisting of one loop that may contain a large number of potentially nested conditional assignments. More complex constructs can then be built atop this basic core, providing a higher-level language that can be used when creating definitions. ASMs can encode any computation and have a rigorous semantics, so any programming language can be defined as an ASM, giving it a semantics. This has been shown by defining non-trivial languages \[88\], such as Java \[165\] and C \[68\], with the Java definition including features such as concurrency and exceptions.

One limitation in the basic ASM language is that support for modular definitions of language features is limited. For instance, the definition for Java separates the languages into named macros based on groupings of features (expressions dealing with threads, expressions dealing with exceptions, etc), joining these together into blocks of rules which are executed based on which language feature is being evaluated. Some modularity is provided by treating the program state like a number of global variables, since this means the state does not need to be explicitly propagated or mentioned in rules that do not use it. However, this may also cause modularity problems – defining a global variable for the environment would be problematic in languages with threads, where a program could have multiple active environments at once, one in each thread.

One way to extend this support for modular definitions is through the use of the Montages \[105\] framework, which combines graphical depictions of language
constructs with static and dynamic semantics notation. Montages also provides a framework for executing programs directly in the semantics. It works by using the control-flow information of a program to “stitch together” the various montages that represent each feature, creating a Montages version of the CFG that will indicate what to run at each step.

**Comparison:** Since ASMs are executable, languages defined using ASM techniques can also be executable, depending on proper tool support. Available tools include ASM Gopher and AsmL [89]. There does not appear to be a common set of higher-level constructs, targeted at multiple ASM tools, which have been developed to support language design, making the process of designing a language using ASMs seem more “low level” than in other formalisms (including MSOS and K).

One of the main uses of abstract state machines has been verification, and at this point the work on verification is more advanced than similar work using K. This includes programming languages-related work on certifying compiler back-ends [194] and verifying runtime properties of .NET code [15]. Some work on model checking ASM definitions has also been done [190] [191], although it is unclear if this work would scale to model checking programs in programming languages defined using ASM techniques.

Specifically for Montages, one limitation (which does not seem fundamental) is that it is not possible to define more than a single static and a single dynamic semantics, making it more challenging to define alternate semantics, such as semantics aimed at program analysis. Another limitation is that Montages uses concrete, versus abstract, syntax, requiring the underlying Montages technology to focus inordinately on compiler-like analysis to correctly “connect” the various Montages in order to properly represent programs in a language. The focus in K and most other formalisms on using abstract syntax instead eliminates the need for this type of analysis, but at the cost of requiring a translation from concrete to abstract syntax.

**Rewriting Logic Semantics**

Beyond the work done on K, Maude has also been used as a platform to experiment with other styles of semantics, enabling the creation of modular language definitions. Some of this work was mentioned above: the Maude Action Tool [38], which provides support for action semantics definitions in rewriting logic; and the Maude MSOS Tool [27], providing support for MSOS definitions. Other work has focused on defining the semantics of a number of different languages and calculi, including BC [21], CCS [184] [21], CIAO [169], Creol [94], ELOTOS [183], MSR [26] [167], PLAN [168] [169], the ABEL hardware description language [98], and the $\pi$-calculus [173].

Some work has focused specific on the modularity of language definitions.
Work on defining Eden \cite{71}, a parallel variant of Haskell, has focused on modularity to allow for experimentation with the degree of parallelism and the scheduling algorithm used to select processes for execution. General work on modularity of rewriting logic semantics definitions \cite{118} \cite{21} has focused on specific definitional guidelines for defining modular features that need not change as a language is extended, a goal similar to our own.

**Comparison:** The work on K is part of the same family as much of the work mentioned here, and also takes advantage of the same tools provided by Maude for working with languages: interpreters, breadth-first search, and model checking. Maude itself does not impose a modular structure on the defined semantics, so many of the languages mentioned above are defined in a style that is not modular, such as by using a definition style similar to that provided by natural or structural operational semantics. Also, K was defined to eliminate some of the challenges we encountered when defining languages directly using Maude. The biggest challenge, the need to modify the configuration as it is reorganized or extended (such as by adding concurrency), has been addressed by context transformers, while the need to duplicate unchanged parts of the term on both sides of a rule or equation has been addressed by using the two-dimensional “contextual” equation and rule style, with changes to a term given by underlining the part that is changing and places the changes below.

**Semantics-Based Interpreters**

In many cases the semantics of a language is defined inside another language by defining a semantics-based interpreter. Many examples of this were given above, especially in the context of monads, where most of the published work has been accompanied by language interpreters written in Haskell, Gopher, or Scheme. The work on K and (more generally) rewriting logic semantics could also be cast in this light, with interpreters based on term rewrite systems derived from the equations and rules used in the definition.

One interesting application of these techniques is in teaching language design and language semantics. Interesting work in defining semantics-based interpreters for teaching includes work on using Scheme to define language interpreters \cite{54} and using Prolog to explore various styles of semantics \cite{164}, including SOS, natural semantics, denotational semantics, and action semantics.

**Comparison:** In both cases, one strength of the approaches, shared by similar techniques using rewriting logic semantics and K \cite{154}, is that the semantics are executable, providing more feedback to students and making the task of language design feel more like the task of programming. On the downside, one drawback of using full programming languages, like Scheme, is that some of the details of language design can get hidden inside the language: designing language features
of the defined language in terms of (especially complex) features of the defining language may conceal the true complexity of a feature instead of making the definition clear.

**Term Rewriting and Equational Logic**

Among the approaches based on term rewriting and related techniques, the first extensive study on defining a programming language equationally [61], was performed using OBJ [59] to execute the language specifications via term rewriting. This is similar to work on defining deterministic sequential languages using rewriting logic semantics, and influenced some of the early work on using RLS techniques to teach programming language design [154].

The ASF+SDF Meta-Environment [179], a successor to Centaur, has focused on program analysis, programming language semantics, and program renovation. The Meta-Environment includes a number of tools designed specifically for working with language definitions, including parsers, pretty printers, and libraries of syntax specifications. Some work has focused specifically on language prototyping [181], with motivations similar to our own. Of special note is the ability to compile definitions to improve performance [180], an important ability for generating realistic interpreters and analysis tools. Use of the Meta-Environment does not impose a specific semantics style (such as SOS), but instead provides a platform where definitions in various styles can be used.

**Comparison:** The work discussed here does not focus on a new style of semantics; instead, it focuses on providing tools that can be used to create language definitions, either for pedagogical purposes, for program verification and analysis, or for language design. The work on using equational logic to define language semantics for imperative programs can be seen as a precursor to the work on rewriting logic semantics and K, which both extend it to support more modular definitions and concurrency. Comparing K with the Meta-Environment, ASF+SDF provides better tool support for language definition (but not necessarily for verification), including the ability to compile definitions, and a larger library of predefined features. On the other hand, the ability in K to use commutative as well as associative matching (ASF is limited to associative matching) provides additional flexibility in defining language features (parts of the configuration can be matched more easily, without regard to order, while unused parts do not need to be mentioned at all), further enhanced by the use of context transformers to automatically modify rules and equations to match the current configuration. We believe this added flexibility allows more modular language definitions to be created while still using algebraic definitional techniques.
Theorem Provers

By being both a (functional) programming language and a theorem prover, ACL2 [99] provides an environment that allows the definition and formal analysis of programming languages. As part of the work on language definition with ACL2, the operational semantics of a substantial subset of the Java Virtual Machine (JVM) has been defined [99]. Similar work has been carried out using other theorem provers. Definitions can be executable, although sometimes only under certain assumptions (functions may need to be total to be usable for theorem proving purposes as well, for instance).

Comparison: While ACL2 gains additional power through the support of the underlying theorem prover, this also makes it more challenging for more “typical” language designers to use as an environment for language prototyping. Also, since ACL2 is inherently sequential, to support concurrency the semantics need to include a formal representation of a thread scheduler, something not required in K. Finally, the use of standard operational techniques for creating language definitions limits modularity, requiring changes throughout the definition when new features are added that modify the configuration. Similar restrictions are present in other theorem provers as well, which generally base language definitions around either standard operational or denotational techniques.

9.2 Program Analysis

The work on policy frameworks, discussed in Chapters 7 and 8, focuses on support for annotation-driven analysis, which is the focus here as well. Because the domain of units of measurement has provided a goal for much of our work on analysis, a section devoted just to related work on units, including systems that do not use annotations, is also provided.

9.2.1 Analysis Tools and Frameworks

A number of different tools and techniques have been developed around the use of annotations for program analysis. The earliest precursor to the work presented here was developed as a prototype to check the unit safety of programs written in BC [31], and was policy and language specific. An extension to this work, C-UNITS [156], is closer in style to CPF, using annotations to check unit safety for a limited subset of C. However, C-UNITS is not extensible, offering no clear way to either support other analysis domains or cover unsupported features of C.

JML [25], the Java Modeling Language, provides support for a wide variety of annotations and a number of different analysis tools, including those used for runtime and static analysis. Spec# [14] extends the C# language with support for a number of annotations, with checking performed by generating verification tasks for theorem provers. Both JML and Spec# are language specific
(Spec# is actually its own language) and would require potentially cumbersome axiomatizations of analysis domains such as units (for instance, by defining units in first-order logic). However, both provide richer support for proving properties about programs that is currently available in the work on policy frameworks, and both have seen extensive use (especially JML, which has been used in a number of different verification tools).

Eiffel [122] uses a design by contract approach to software development, including direct language support for annotations such as preconditions (require) and postconditions (ensure). This support is obviously language specific, but because of this is very well integrated into the language.

Specifically for C, a number of annotation-based systems have been developed. LCLint [47, 46], now Splint, uses program annotations to detect potential errors in C programs, and provides limited abilities to add new annotations by allowing attributes and constraints to be defined for various C language objects. Splint is faster than CPF, but CPF provides a more flexible (and easily extensible) annotation language. Caduceus [51, 52] provides an annotation language similar to JML, but usable in multiple languages (currently C and Java); programs are verified by transforming them into a simpler language, called Why, which is then further processed to generate proof tasks for various theorem provers. While the annotation languages defined in policy frameworks are more easily extended, the reduction of annotations and language constructs in multiple languages into a single core language provides a level of reuse worth investigating in more detail for policy frameworks, which give an analysis semantics for each language.

Another solution for C is Frama-C [1], which provides an extensible analysis framework, with various analyses built in OCaml as “plugins” to the core Frama-C tool. Frama-C uses the ACSL annotation language [16], which is based on the annotation language used in Caduceus, and which can also be extended to support new logical concepts. Frama-C has many similarities to CPF, including the use of CIL, but (like in many of the above systems) the requirement to formalize extensions in first order logic makes it harder to use Frama-C for verification of domain-specific properties such as units.

More domain-specific systems include VCC [34], for verification of concurrent C programs, and HAVOC [29], aimed at programs, such as device drivers, that perform low-level memory manipulation. Both of these systems are targeted at specific domains, and it is not obvious how to extend them to other domains or languages. CQUAL [53] provides support for user-defined type annotations, referred to as type qualifiers, and has the added benefit of being able to infer many qualifiers; however, in CQUAL it is hard to natively support some complex domains, such as units, which (as discussed below) leads to the use of more complex solutions.

The analysis techniques used in policy frameworks have many similarities to abstract interpretation [35]. In fact, the analysis could be seen as an abstract interpretation, especially (in the case of units) over an enriched concrete domain.
(enriched enough to allow sensible abstraction and concretization functions to be given). An interesting goal of future research would be to investigate this link further.

9.2.2 Units of Measurement

Related work on unit safety tends to fall into one of three categories: library-based solutions, where libraries which manipulate units are added to a language; language and type system extensions, where new language syntax or typing rules are added to support unit checking in a type checking context; and annotation-based solutions, where user-provided annotations assist in unit checking.

Library-based solutions have been proposed for several languages, including Ada [72, 112], Eiffel [100], and C++ [24]. The Mission Data Systems team at NASA’s JPL developed a significant library, written in C++, which includes several hundred classes representing typical units, like MeterSecond, with appropriately typed methods for arithmetic operations. An obvious disadvantage of such an explicit approach is that the units supported by the library are fixed: adding new units requires extending the library with new classes and potentially adding or modifying existing methods to ensure the new classes are properly supported.

Solutions based around language and type system extensions work by introducing units into the type system and potentially into the language syntax, allowing expressions to be checked for unit correctness by a compiler or interpreter using extended type checking algorithms. MetaGen [9], an extension of the MixGen [8] extension of Java, provides language features which allow the specification of dimension and unit information for object-oriented programs. Other approaches making use of language and type system extensions have targeted ML [102, 101], Pascal [55, 87], and Ada [56]. One major limitation of these types of systems is that users are often not willing to move to a custom version of a language, but must stay with the original, even if the custom version has some important benefits. This is one of the reasons that CPF has focused on allowing annotations to be added in comments, where they can be ignored by standard C compilers.

A newer tool, Osprey [93], also uses a typed approach to checking unit safety, allowing type annotations in C programs (such as $\text{meter int}$) using a modified version of CQUAL. These annotations can then be checked using a combination of several tools, including the annotation processor, a constraint solver, a union/find engine, and a Gaussian elimination engine (the latter two used to reduce the number of different constraints and properly handle the Osprey representation of unit types as matrices). One limitation of Osprey is that there is no way to express relationships between the units of function parameters and return values, something possible with a richer annotation language:

```c
//@ post(UNITS): unit(@result)^2 = unit(x)
double f(double x) { ... }
```
Instead, this type of relationship has to be added by hand-editing files generated during processing. Osprey also checks dimensions (i.e., length), not units (i.e., meters or feet), instead converting all units in a single dimension into a canonical unit. This can mask potential errors: for instance, it is not an error to pass a variable declared with unit meter to a function expecting feet. On the other hand, Osprey includes functionality to check explicit conversions for correctness, catching common conversion errors such as using the wrong conversion factor.

Annotation-based systems for unit safety include the work on C-UNITS [156], and BC [31], both mentioned above. The CPF UNITS policy was inspired by the work on C-UNITS, and takes a similar approach, with a focus on using abstract semantics and annotations. However, CPF UNITS has extended this approach in three significant ways. First, it has been designed to be modular: the abstract semantics of C have been completely redefined using concepts developed over the last several years as part of the rewriting logic semantics project and the work on K. As described in Chapter 8, the semantics are divided into core modules, shared by all CPF policies, and units modules, specific to CPF UNITS. This allows improvements in the core modules to be shared by all policies, simplifies the unit checking logic, and greatly improves the ease with which the semantics can be understood and modified. Second, CPF UNITS has been designed to cover a much larger portion of C. C-UNITS was designed as a prototype, and left out a number of important C features, with minimal or no support for structures, pointers, casts, switch/case statements, gotos, or recursive function calls. Support for expressions was also limited, with the main focus on commonly-used expressions, and more complex memory scenarios (structures with pointers, arrays of pointers, etc) were ignored. CPF UNITS supports all these features, and makes use of a more advanced parser to handle a larger class of C programs. Finally, CPF UNITS has been designed to be more scalable. While C-UNITS requires a complete program for analysis, the CPF UNITS policy analyzes individual functions, leading to smaller individual verification tasks.
Chapter 10

Conclusions and Future Work

As discussed in Chapter 1, software is now a pervasive part of our lives. Computer programs handle bank transactions, calculate driving directions, and control small embedded systems present in phones, cars, airplanes, and household appliances. While formal techniques provide a firm mathematical basis for understanding these programs, and the languages used to create them, they often are considered too cumbersome to use. It is thus important to have formal, yet flexible, tools and modeling techniques, which ideally should be easily understood yet powerful enough to tackle realistic problems.

Chapter 2 presented background on techniques that we believe meet these goals: term rewriting, rewriting logic semantics, and K. Term rewriting provides an easily understood yet powerful model of computation; rewriting logic semantics and K provide the flexibility needed to formally model languages with complex features. Of special interest are features for complex control flow and concurrency, both of which have traditionally caused problems for formal semantics techniques, but both of which are important in real programming languages.

Chapters 3 and 4 showed how these techniques can be used for programming language design. Chapter 3 showed an approach to language design based around language prototyping, taking advantage of the flexibility of the underlying semantics to provide an environment for creating new language features and immediately testing them on real programs. Although the KOOL language was used to provide a motivating example, these techniques can also be applied to other languages, something shown in Chapter 4 by applying these techniques to the Beta programming language.

One point not addressed in Chapters 3 and 4 was the ability to package features into reusable units. This was addressed in Chapter 5, which presented the K module system. The module system provides important functionality for K, allowing definitions to be spread over multiple modules, combined into complete languages, and reused in the definitions of new languages. It also provides important tool support, including the ability to generate modules for the K tool support developed in Maude and to retrieve modules from a shared online module repository.

Another point not addressed in these earlier chapters is performance. With executable specifications and a prototyping approach to language design, perfor-
mance can become quite important. Performance is also important for program analysis, ensuring that analysis results are computed quickly. Chapter 6 addressed performance in the contexts of both program evaluation and program analysis, with a special focus on definitions of memory and the representation of program values in a pure object-oriented language.

This led directly into program analysis, which provides another way to leverage created language definitions. Chapter 7 described the concept of policy frameworks, presenting an example using the SILF language. To show that this concept is not limited to simple, purpose-built languages, Chapter 8 presented the C Policy Framework, or CPF, which applied the policy frameworks concept to C. Two policies, one for memory safety and one for units of measurement, were presented as part of the work on CPF. These policies illustrated how the framework could be extended to handle a new analysis domain. The units of measurement policy also showed that it is possible to create analysis tools to outperform competitor tools in some areas (flexibility of annotation language, ability to find errors) while remaining competitive in others (performance).

**Future Work**  Along with the work already described there is much future work yet remaining. For language prototyping, improving tool support, defining new languages, and defining complex language constructs, all parts of the current research, should be continued. For policy frameworks, extending the concept to new languages and new domains, as well as extending existing tools to handle more complex requirements (e.g., some aliasing requirements in C), are both areas of possible research, as is focusing more on using the policy frameworks infrastructure to facilitate verification using third-party theorem provers. The links between policy frameworks and abstract interpretation, as well as the ability to transform language constructs into an intermediate language for policy analysis (similar to the Caduceus system mentioned in Chapter 9), are also interesting ideas that should be investigated.

The research on modularity is the newest at this point. The module system for K is still relatively young, so it is important to apply it to many real programs, using the feedback from this process to improve the system further. Other work, more focused on tool support, is needed to improve up-front parsing of modules (needed for front-end tools, like plugins for Eclipse or NetBeans), visualization of modules and module operations, and interaction with the module repository. At the same time, a version of the module system built around an extension of Full Maude is currently being developed, providing an alternative for people more familiar with working directly in that environment. As the repository is intended to support formalisms beyond K, it is also important to extend work on the repository and the XML module exchange format to comfortably handle other formalisms, allowing (for instance) for MSOS modules, RLS modules, Action Semantics modules, etc to take advantage of the repository for module reuse.

At a more foundational level, it is important to develop a clearer understand-
ing of the theoretical underpinnings of the module system. The current planned
approach is to base this formalism around institutions [60], an approach taken
previously to formalize modules with information hiding [62] (a requirement for
the K module system) and to formalize the Maude module system [43, 12].
References


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