Algebraic Considerations

- Show f:ú6ú $^{+}$ defined by $f(x) = e^{x}$ is a bijection
  - Note: ú $^{+}$ refers to the positive real numbers
- Let's show f is 1-1:
  - Suppose $f(x) = f(y)$. Then
  - Taking the natural log of both sides: $\ln e^{x} = \ln e^{y}$
  - which implies $x = y$
- Let's show f is onto:
  - Select y0ú $^{+}$. We wish to find x0ú such that $f(x) = y$
  - Solve $e^{x} = y$ by taking the natural log of both sides
  - $\ln e^{x} = \ln y$ 6 $x = \ln y$,
  - (we can take the natural log of any positive real number)
- Thus f is a bijection

A Note on Proving a Function is 1-1

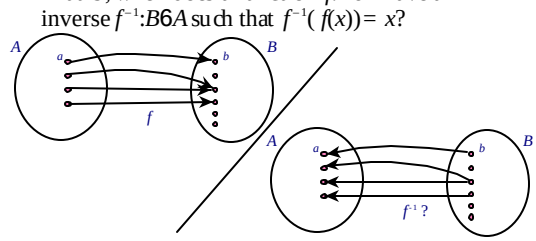
- Here is our original definition of a function being 1-1:
  - f:A6B is 1-1 if x...y 6 $f(x)$...$f(y)$
- An equivalent definition uses the contrapositive of our definition (in red) above:
  - f:A6B is 1-1 if $f(x)$=$f(y)$ 6 x=y
- Since a proposition is logically equivalent to its contrapositive, we may use the contrapositive as our definition for 1-1.
- This is what we did on the preceding and following slides.

Your Turn!

- Show that f:Z6Z defined by $f(x) = 5 - x$ is a bijection
  - We need to show that f is 1-1 and onto
- Let's show that f is 1-1
  - Suppose that $f(x) = f(y)$.
  - Then $5 - x = 5 - y$, which implies $x = y$.
- Let's show that f is onto
  - Let y be some element of Z
  - We want to find some element x of Z such that $f(x) = y$.
  - Solving $5 - x = y$, gives $x = 5 - y$, and x 0 Z.
- So f is a bijection.

Inverse of a Function

- Under what circumstances does a function have an inverse?
- That is, when does a function f:A6B have an inverse f^{-1}:B6A such that $f^{-1}(f(x)) = x$?



Inverses and Bijections

- A function $f{:}A6B$ has an inverse $f^{-1}{:}B6A$ if and only if f is a bijection
- On the other hand, suppose you were willing to have an inverse defined not on all of B, but on a subset of B
  - Which subset?
  - The *Range* of f
- Under what circumstances can you find an inverse by restricting to the range?
  - If the function is 1-1

Sequences

- A sequence is an ordered collection of objects
- It's a tuple without the parentheses
- but it can be finite or infinite
- Sequences do not always start with element 1; they can start with their 0th element, or 2nd element, etc...
- Here is a five element sequence:
  - a_1, a_2, a_3, a_4, a_5
- Here is an infinite sequence:
  - $a_2, a_3, a_4, a_5, \ldots$

Formula for a Sequence

- Sometimes sequences are given by a formula:
  - $a_k = k^2 - 2$, for $k \$ 1$
  - sequence: –1, 2, 7, 14, ...
  - It's an infinite sequence
- And sometimes we are given a sequence and are asked to find a formula for that sequence:
  - 1, 3, 6, 10, 15, 21, 28, 38, 45, 55, ...
  - But that's a topic for another day

Sum of a Sequence

- Sometimes we're going to want to add together the terms of a sequence
- If $a_1, a_2, a_3, a_4 \ldots, a_k$ is a sequence, then we write the sum of this sequence as:
  - $a_1 + a_2 + a_3 + a_4 + \ldots + a_k$

$$= \sum_{i=1}^{k} a_i$$

Terms to be added

Ending value

$$\sum_{i=1}^{k} a_i$$

Index and starting value

Your Turn

Evaluate: $\sum_{k=1}^{5}(k^2+1)$

Evaluate: $\sum_{k=1}^{6}(k^2+1)$

Evaluate: $\sum_{j=1}^{3}\sum_{k=1}^{4}(k^2+j)$

Evaluate: $\sum_{j=1}^{5}\sum_{k=1}^{j}(k^2+1)$