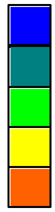
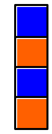
Arranging Distinct Objects

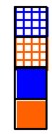


- Question:
  - How many ways are there to stack the blocks shown to the right?
- Answer:
  - There are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ = 120 ways to build such a stack
- In General:
  - The number of ways to arrange *n* distinct objects is $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$

Arranging Objects with Repeats



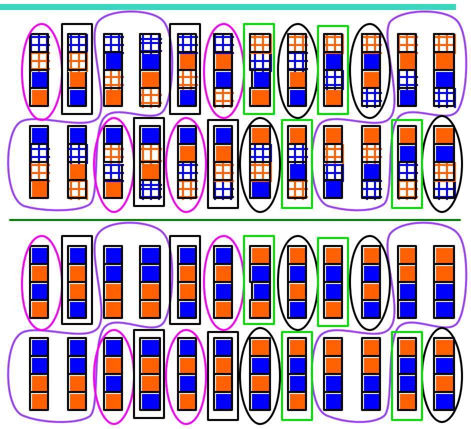
- How many ways are there to arrange the objects on the right?
- The answer is not 4!, because the objects are not all distinct
- We need a way to deal with repeats
- Our trick is to:
  - temporarily make them all distinct
  - then correct for overcounting



- With distinct colors, as shown here, there will be 4! = 24 ways to arrange the objects

Arranging Objects with Repeats

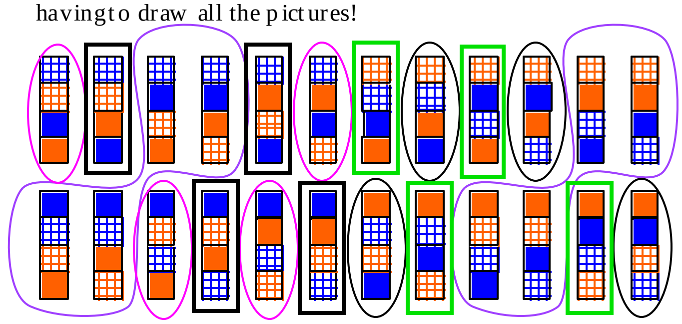
- Here are the 24 ways with distinct colors
- They are not all distinct with the original colors



Predicting the Number in each Group

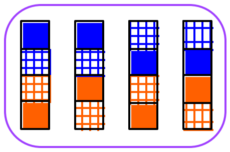
How could we have predicted that each group would have 4 arrangements in it?

We would then know that 24/4 = 6 was the answer without having to draw all the pictures!



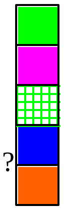
Inside the Purple Group

- Here is a redrawing of the purple group
- How could we have predicted that there would be 4 arrangements inside this group?
- There are two "blue" blocks.
  - They can be arranged in 2! ways.
- There are two "red" blocks.
  - They can be arranged in 2! ways.

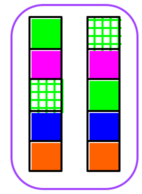


- So this group will contain $2!\times 2! = 4$ arrangements (by the product rule)
- Similarly, every group will contain 4 arrangements

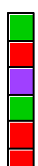
How Many in each Group?



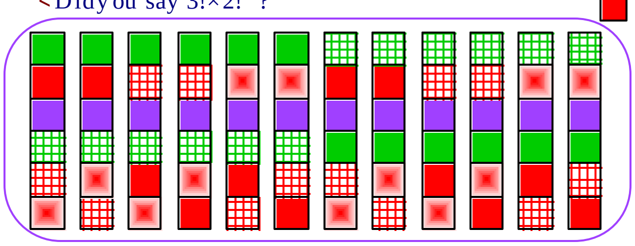
- Here is another tower of colored blocks
- Suppose we assign different colors to each block. How many arrangements?
- If we regroup those that are really the same, how many arrangements will be in each group?
- There will be 2! in each group, because there are 2 greens, which are really the same.



How Many in each Group?



- Same question for the tower of blocks to the right:
  - When we assign distinct colors and then regroup, how many will be in each group?
  - Did you say $3!\times 2!$?



Your Turn

- How many "anagrams" are there of each of the following words?
  - READ
    - $4! = 24$
  - REED
    - $4! / 2! = 12$
  - DEED
    - $4! / (2! \times 2!) = 6$
  - EPEE
    - $4! / 3! = 4$
  - ANAGRAM
    - $7! / 3! = 840$
  - SORTING
    - $7! = 5040$

Applications of the Anagram Method

- Computing the "choose numbers."
- Given *n* distinct objects, the number of ways to select *k* of them is called "*n* choose *k*"
- For example, 6 choose 2 = 15, because there are 15 ways to select 2 objects from among 6 distinct objects
  - If we wish to select two letters from "ABCDEF", we could select AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, or EF. That's 15 ways.
- We will now derive a better way to do it.

Selecting 2 out of 6

- Let's line up the six letters "ABCDEF" and consider some ways to select two of them

A B C D E F	Selection
Y Y N N N N	AB
Y N N N Y N	AE
N N Y Y N N	CD
...	...

- Place a "Y" under each letter that is chosen, and an "N" under each that is not chosen
- Each way of selecting two letters corresponds to an anagram of the word "YYNNNN"
- Thus, the number of ways to select two letters is 6!/(2!×4!) = 15

Selecting 4 students out of 10

- A teacher wishes to select 4 students from a class of 10. In how many ways can this be done?
- Line up the students: A_B_C_D_E_F_G_H_I_J
- Consider a way to select 4: Y Y Y Y N N N N N N
- Again, we see that every way to select 4 students corresponds to an anagram of "YYYYNNNNNN"
- So there are 10!/(4!×6!) = 210 ways to do this
- In general, the number of ways to select *k* objects from *n* distinct objects is $n!/(k! \times (n-k)!)$

Walking on a Grid

- Another application of anagrams:
- How many ways are there to walk from A to B on the grid below, walking only north and east?
- One way to walk: NNNNNNEEEE
- Another way: EEEENNNNNN
- Another way: NNEENNEENN
- All ways to walk have 4 "E" and 6 "N"
  - And are thus anagrams of "NNNNNNEEEE"
- The number of ways to walk is thus:
  - 10!/(4!×6!) = 210