## Homework Review

We built the truth table for $((p \to q) \to r) \to s$

| $p$ | $q$ | $r$ | $s$ | $p \to q$ | $(p \to q) \to r$ | $((p \to q) \to r) \to s$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | T | F | T | T | F |
| T | T | F | T | T | F | T |
| T | T | F | F | T | F | T |
| T | F | T | T | F | T | T |
| T | F | T | F | F | T | F |
| T | F | F | T | F | T | T |
| T | F | F | F | F | T | F |
| F | T | T | T | T | T | T |
| F | T | T | F | T | T | F |
| F | T | F | T | T | F | T |
| F | T | F | F | T | F | T |
| F | F | T | T | T | T | T |
| F | F | T | F | T | T | F |
| F | F | F | T | T | F | T |
| F | F | F | F | T | F | T |

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## Logical Symbols

| | | | |
|---|---|---|---|
| Negation | $\neg p$ | not $p$ | True when $p$ is false |
| Conjunction | $p \wedge q$ | $p$ and $q$ | True when both $p$ and $q$ are true |
| Disjunction | $p \vee q$ | $p$ or $q$ | True when either $p$ or $q$ is true, or both |
| Implication | $p \to q$ | $p$ implies $q$<br>if $p$, then $q$<br>$q$ only if $p$<br>*implicitly* | Always true, unless $p$ is true and $q$ is false. |

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## Compound Statements

- A statement that is composed of simpler statements joined by some logical connective(s) such as "$\wedge$," "$\vee$," "$\oplus$," etc... is called a *compound proposition*
- A statement which is not a compound statement is called a *simple proposition*
- We are thus interested in determining the truth value (True or False) of a compound proposition given the truth values of the simple propositions of which it is composed.

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## Truth Tables

| $p$ | $\neg p$ |
|---|---|
| T | F |
| F | T |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

This is the *exclusive OR*

This is the *biconditional*

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

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## Compound Truth Tables

We use truth tables to compute the truth values of our compound propositions.

- $(p \vee q) \wedge (p \oplus q)$
- $(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$
- $(p \rightarrow q) \vee (p \wedge q)$

This was our first group quiz



## Logical Operations on Bits

- Note that we can compute the value of arbitrarily complex logical formulas with just a few truth tables
- This idea is the foundation of digital computing
  - Each little logical operation is easy for an electronic circuit
  - And we can combine these easy electronic circuits to perform very complex tasks, such as adding two numbers, converting a number from base 10 to binary, factoring large numbers, digital image editing, etc...
- We simply think of "1" as True and "0" as False



## Logical Operations on Bits

| $p$ | $\neg p$ |
|---|---|
| 1 | 0 |
| 0 | 1 |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |



## Some Notes on the Implication

- Let $p \rightarrow q$ be an implication
- Then $q \rightarrow p$ is called the *converse* of $p \rightarrow q$
- And $\neg q \rightarrow \neg p$ is called the *contrapositive* of $p \rightarrow q$
- As we shall see, the contrapositive of an implication is equivalent to the implication itself. This is a very important and useful observation when we wish to do proofs.