Discrete Mathematics

for computer science

Robert Hochberg, T-Th 9:30-10:50, Austin 304

Text: Discrete Mathematics and its Applications
by Kenneth Rosen, 4th edition

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Who are YOU?

Please write your name on the index card.

Place it in the hat.

Select one at random when the hat comes around

Find the person on the card you selected, and find out that person's:

Name, Major, class Year and what they think Discrete Math is.

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Permutations

- Every permutation can be decomposed into its component cycles
- What is the expected number of fixed points?
- What are the chances that Chris and Bobby are in the same cycle?
- What is the expected number of cycles?
- What is the most likely cycle structure?
- What is the least likely cycle structure?

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Permutations

- Why can't *** win the 8 puzzle?
  - Is the puzzle really difficult?
  - Is the puzzle solver a bit dim?
  - Is the puzzle *impossible*?
- Could Einstein solve the puzzle?

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Logic

- The graph G is not planar
- All the vertices of G have degree 3
- Every non-planar graph contains a subdivision of either K_5 or $K_{3,3}$
- If a graph contains a subdivision of K_5, then it must have at least five vertices of degree at least four

Prove that G contains a subdivision of $K_{3,3}$

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Proof

Since G is not planar, we know that it must contain a subdivision of either K_5 or $K_{3,3}$. Since all the vertices of G have degree 3, we know that G cannot contain any vertices of degree at least 4. But then G cannot contain a subdivision of K_5. Thus G must contain a subdivision of $K_{3,3}$.

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Propositions in Logic

- A *proposition* is a sentence which is unambiguously true or false.
- Propositions:
  - $4 = 2 + 2$
  - $5 > 10$
  - Montana is the capital of Mozambique
- Not Propositions:
  - $x = 21$
  - $\alpha > \beta$
  - The graph G is Eulerian, but not Hamiltonian

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Representing Propositions with Letters

- When discussing the rules of logic, we often denote propositions with letters, just like variables in mathematics
- We can pull out some propositions from what we were given and what we wished to prove:
  - p: G is planar
  - q: G contains a subdivision of K_5
  - r: G contains a subdivision of $K_{3,3}$
  - s: G has at least five vertices of degree at least 4

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Translating to Mathematics

- Our *givens* are:
  - $\neg p$
  - $\neg s$ (this is *implied* by the second given proposition)
  - $p \rightarrow (q \vee r)$
  - $q \rightarrow s$
- We are asked to prove:
  - r
- We can prove this using the mathematical rules of logical inference

But first... A little notation

Logical Symbols

Negation	$\neg p$	not p	True when p is false
Conjunction	$p \wedge q$	p and q	True when both p and q are true
Disjunction	$p \vee q$	p or q	True when either p or q is true, or both
Implication	$p \rightarrow q$	p implies q if p, then q q only if p *implicitly*	Always true, unless p is true and q is false.